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Photo by Pai Liu. Photograph of The University of Texas at Austin's tower from Gearing Hall. Pai Liu is a current Senior Editor for *The Developing Economist* and has been a member of the editorial staff for three years.

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Inquiries should be addressed to:

developingeconomist@gmail.com
Omicron Delta Epsilon
2225 Speedway
BRB 1.116, C3100
Austin, Texas 78712

www.developingeconomist.com

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Thank You,

Editorial Team

The Developing Economist

An Undergraduate Journal of Economics

Dear Reader:

I am excited to present the fifth edition of *The Developing Economist*. The journal has undergone many changes and improvements throughout our operational history and the following publication is the result of the time and efforts of many dedicated and hardworking students eager to contribute to and take part in economics research.

Undergraduate research as a whole has grown exponentially since *The Developing Economist* first started publication in 2014, and it has been thrilling to be able to contribute to this growth.

The following seven papers brilliantly display the drive and intellectual curiosity of the undergraduate student, with each author exerting considerable effort into bringing their ideas and insights to life.

I am thankful to both their efforts and the efforts of the editorial staff in making this journal possible each year. It has been a very rewarding experience to work on *The Developing Economist* during my undergraduate career and I look forward to seeing the continued evolution and growth of the journal as it continues to serve as an important vehicle for undergraduate research.

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Moral Sentiments and Altruism in Game Theory

Joshua Liebmann

Advisor: Dr. Terry Olson

Abstract

This paper presents a model that heavily incorporates key themes from Adam Smith's moral philosophy in order to construct a utility function that takes into account both the utility of others and appeals to fairness. This model is constructed specifically for the purpose of explaining altruistic behavior in the dictator and ultimatum games. The resulting utility function yields an outcome where rational, utility-maximizing individuals willingly offer a considerable portion of money to the other player in order to maximize their overall personal satisfaction.

I. Introduction

In 1759, Adam Smith wrote a work giving a depiction of human nature contrasting sharply with the one often associated with the field that he was instrumental in creating. In *The Theory of Moral Sentiments*, Adam Smith describes human beings as naturally empathetic, and possessing within themselves an inscribed sense of justice and a natural desire to see that justice is realized. This work, written 17 years before his better-known and far more influential work, *The Wealth of Nations*, can initially appear antithetical to the pre-suppositions of modern economic theory that was born out of his later work.

Over the past two centuries, economic analysis has displayed the power and reliability of the *homo economicus* perspective of human nature. Despite its virtues however, there appears to be various phenomena demonstrated in the behavior of humans, and even to some extent animals, that seem to suggest that there are limits to this assumption. These include kindness, acts of service, gifts, honesty, and courtesy. Such

occurrences prove especially problematic when evaluating the assumption of self-interest. This apparent contradiction has even led some to question the credibility, or at least the usefulness, of classical economic theory. This paper proposes that the incorporation of Smith's description of human nature in his former work can account for the vast majority of acts considered to be selfless or altruistic while remaining under the assumption of self-interest. Specifically, the following analysis focuses on behavior in the dictator and ultimatum games that often appears to be the product of altruistic intentions.

In the dictator game, an individual playing the role of the dictator decides how to split a sum of money between himself and another individual. There are no additional limitations or costs associated with the choice made by the dictator, and the other individual has no influence over the outcome. Due to the absence of virtually all external consequences associated with potential outcomes, the dictator game proves particularly useful in the inquiry into the nature of altruistic behavior. Due to the assumed rational and self-interested nature of the dictator, traditional economic theory would suggest that the dictator would allocate the entire sum to himself, leaving nothing for his companion. This result, however, is by no means universal, as dictators in experimental settings have often given a small portion of the total sum (Hoffman et al. 1994; Bohnet and Frey 1999). In the ultimatum game, one player performs the same function as the dictator, but the other individual chooses whether to accept or reject the proposed allocation. Accepting the amount would lead to the same result, as would be the case if the dictator had proposed that split, while a rejection results in a payoff of zero for both individuals. While the ultimatum game does pose potential consequences for excessively selfish behavior, the advantage of using such strategic situations is to investigate more realistic situations in which both individuals principally concerned have some influence over the final outcome. Traditional economic theory predicts that the final outcome would be the first player offering the other an arbitrarily small, yet positive portion and the second individual accept-

ing the offer since it still yields the highest payoff available to him. This prediction too, proves faulty as the second player rejects many nonzero offers that are deemed to small, resulting in substantial offers being made. This serves as the starting point for this paper and is the pattern accounted for. These or similar concepts, techniques, and models may prove useful in providing plausible accounts for instances of altruism and cooperation in more complicated or strategic circumstances.

II. Literature Review

The nature, origins, and causes of altruistic behavior in humans, as well as animals, have become a topic of much interest and inquiry among social scientists, biologists, and philosophers. Research on altruism typically takes one of three forms: a philosophical inquiry into morals and human nature, an experimental and empirical analysis of human behavior in various strategic and non-strategic settings, or a construction of a theoretical model that attempts to explain the origins or causes of altruistic behavior.

In 1759, Adam Smith wrote his first major work, *The Theory of Moral Sentiments*. Within its pages, he laid out his moral philosophy, which places a high value on one's ability to empathize with those around him, and behave in a manner that makes it easier for others to empathize with him. These principles are expressed in a variety of ways, such as by proposing that individuals ought to lower the intensity of their expressed joys and sorrows so others may find it easier to empathize with them, or by saying we ought "to love ourselves no more than our neighbor can love us." Throughout his work, he provides justifications that begin to depict his view of human nature; a being that instinctively empathizes with others while consciously operating on the principle of self-interest. Such an illustration suggests that altruistic behavior may be able to be explained, while maintaining the assumption of rational, self-interested behavior characteristic of economic theory.

As previously stated, *Moral Sentiments* provides many concepts that may be employed to account for altruistic behavior

and cooperation in a variety of contexts. Heath (1995) discusses the application of Adam Smith's moral philosophy to give a possible account for the emergence of morals, while also discussing its similarities and dissimilarities with the moral philosophy of David Hume. Evansky (2005) analyzes a number of key themes in *Moral Sentiments* in a more political context arguing that Smith's philosophy presents groundwork for revealing how society has been able to so effectively avoid the Hobbesian state of nature. Ashraf, Camerer, and Loewenstein (2005) discusses the relevance of Adam Smith's first major work with respect to the investigation into the causes of altruistic behavior, particularly the crucial role of the impartial spectator as the standard to judge the actions and motives of others. They go on to discuss various less well-known concepts and excerpts from *Moral Sentiments* regarding self-sacrifice, as well as people's tendency to empathize more with those whom are better off and to attempt to ignore the sufferings of the less fortunate. Hilbe (2009) discusses the effects of mistakes made by individuals in regards to each player's perception of the given situation and mistakes in the implementation of strategies. Hilbe's analysis leads him to conclude that, due to such errors, feelings of guilt are not sufficient for ensuring sustainable cooperation in the iterated prisoner's dilemma game.

In empirical settings, there seems to be a nearly unanimous consensus that the assumptions made in economics have overlooked some aspect that can have significant influence upon people's behavior. While the deviation from expected behavior varies in magnitude among experiments, the results justify a reevaluation of the approach typically taken in economics, and game theory in particular. Perhaps the most near-to-earth and relevant of these studies is that of Monroe, Barton, and Klingemann (1990), which discusses interviews with individuals that lived in Nazi Germany who did and did not assist Jews escaping Nazi persecution, as well as a sample of entrepreneurs. They investigated possible selfish motives that could possibly explain the behavior of those who provided protection that was observed in the latter two groups. These plausible expla-

nations included psychic goods, possible clusters of altruists, and the consideration of altruism to be a luxury enjoyed by the upper classes. After conducting the interviews, they concluded that none of the proposed egoistic motives could accurately account for the benevolent actions and concluded that instances such as these demonstrated the existence of a significant limitation to traditional economic theory.

There is an abundance of literature discussing the results of variants of the dictator game. Eckel and Grossman (1995) reports the results of a dictator game experiment nearly identical to the one found in Hoffman, McCabe, Shachat, and Smith (1994). In both cases, participants playing the role of the dictator decided how to split \$10.00 between themselves and another. After combining the results from the two experiment, providing a total of 48 observations, they found that 62.5% of dictators gave themselves the entire sum, while roughly 8.33% gave at least half away to their companion. They followed this with a dictator game in which they split money with a charity instead of another person. In this case, about 27.1% kept all the money and 31.25% gave away at least half of the pie. While these results demonstrate the average reliability of the principle of self-interest, it does justify inquiry into possible exceptions or complications that may arise in such situations. Burks, Youll, and Durtschi (2012) describes and discusses an experiment designed to empirically observe the association between empathy and altruism. They used the Balanced Emotional Empathy Scale and the Self-Report Altruism Scale in order to take measurements of the individuals' empathy and altruistic behavior respectively. They found a significant and positive relation between the two with an R^2 value of roughly 0.24. Bohnet and Frey (1999) discusses the results of an experiment in which subjects were randomly paired and assigned to play complementary roles in the dictator game under various conditions. These conditions were two-way identification, one-way identification, one-way identification with information, and entirely anonymous. Only under two-way identification were the majority of shares equal.

The next largest proportion of equal shares took place under one-way identification. The curious result was that the games which took place under anonymity had a higher percentage of equal shares than those which took place under the one-way identification with information.

In addition to the dictator game experiments above, Koch and Normann (2005) reports an experiment in which two tests were conducted in order to determine if the dictators gave these large portions away to acquire a favorable or avoid an unfavorable opinion of the other individual. To achieve this, they conduct one experiment in which the passive player in the dictator game is aware of their involvement in the experiment while in the second, individuals playing this role are altogether unaware of their involvement in the experiment. Individuals of this second kind simply received the sum of money allocated to them by the dictator without any knowledge of where it came from. This was done to ensure that the dictator would have no hope of receiving any favorable or unfavorable regard from the other individual involved. The results between the two experiments had no statistically significant difference, leading the authors to conclude that dictators are not motivated by regards for social norms or fairness when deciding how to divide the funds between the two individuals principally concerned.

The literature also displays a number of results of various experimental ultimatum games. Oosterbeek, Sloof, and Van de Kuilen (2003) report the results of experimental ultimatum games conducted in 26 countries throughout the world. The mean offers and mean rejection values vary considerably among the different countries, with the average offers varying between being 26.00 and 51.00, while the average rejection values varied between 0.00 and 33.50 respectively. While these values may depend in part on the standards and preferences of the individual participants, especially since there were frequently under 5 observations per country, it does raise some questions regarding the influence that one's culture or surroundings have on their degree of generosity and preferences in general. A similar experiment analyzed in Henrich, Boyd,

Bowles, Camerer, Feht, Gintis, and McElreath (2001) compared the results of an ultimatum game experiment conducted in 15 small-scale countries. The results also varied considerably, further justifying the suspicion that cultural values play a role in the level of generosity displayed in an individual's behavior.

The above works are only a sample of a growing literature, all drawing similar conclusions. In response, academics throughout the social sciences have begun to develop theoretical models that better display the true behavior and preferences of humans in both strategic and non-strategic settings. Cox, Friedman, and Gjerstad (2006) construct a model that accounts for the unexpected generosity by considering the role that hopes of potential reciprocity plays in determining how an individual interacts with those around him. Their model also considers the impact that the marginal rate of substitution between the utility of the actor and the other individual involved will have of the results of the game. Podkopaev constructs a model that offers a way to calculate altruistic equilibrium, though he does not make any explicit use of Smith's moral philosophy. Kaplow and Shavell (2007) develops a model investigating the impact that the instillation of guilt and virtue have on society as well as a method of measuring the associated costs. One result of their analysis suggests that guilt alone, while useful, proves insufficient in preventing immoral or criminal behavior in a real-world context.

III. Utility and *The Theory of Moral Sentiments*

The model and approach presented in this paper make extensive use of the concepts discussed by Adam Smith in *Moral Sentiments*. The concepts that are incorporated include sympathy, approbation and disapprobation, and the impartial spectator. The objective of this section is to apply the discussed concepts to provide a more complete account and description of the nature and formation of an individual's utility in a particular set of circumstances.

Adam Smith begins *Moral Sentiments* by asserting that sympathy (or empathy) is an intrinsic and universal component of human nature.

How selfish soever man may be supposed, there are evidently some principles in his nature, which interest him in the fortune of others, and render their happiness necessary to him, though he derives nothing from it except the pleasure of seeing it.
(Smith, 1759)

According to Adam Smith, humans naturally adopt the emotions experienced by those around them, albeit at a considerably lower intensity. However, because people cannot actually know with complete certainty what others are thinking or feeling, the only way that one can assess what their neighbor is feeling is by imagining what they themselves would feel in that circumstance. It is worth noting that the onlooker empathizes with the emotions that they would experience if they were in the place of the observed individual. Hence, when observing another in physical pain, they empathize not with the physical pain itself (i.e.: physically experiencing the pain they presume the other feels), but with the distress that accompanies that pain. Smith also establishes that while the spectator will empathize with the emotions of the individual principally concerned, the spectator's empathetic emotions will never reach the same degree of intensity and vivacity as those experienced by the observed individual. Therefore, individual i empathizes with individual j by considering the level of utility i would experience given j 's payoff x_j (i.e.: finding $U_i(x_j)$) and modifying it according to i 's empathy level for j , denoted $\Gamma_{i,j}$. The value of $\Gamma_{i,j}$ is determined by a number of elements, which will be specified in the following section.

Another important concept discussed early on the *Moral Sentiments* is the concept Smith refers to as "the pleasure of mutual sympathy." This refers to the tendency of individuals to experience pleasure merely from seeing others have the same emotional reactions to certain stimuli as experienced by themselves, in regard to both kind and intensity.

But whatever may be the cause of sympathy, or however it may be excited, nothing pleases us more than to observe in other men a fellow-feeling with all the emotions of our own breast; nor are we ever so much shocked as by the appearance of the contrary. (Smith, 1759)

Smith later ends his discussion on mutual sympathy considering the case of unequal emotional reactions, saying:

On the contrary, it is always disagreeable to feel that we cannot sympathize with him, and instead of being pleased with this exemption from sympathetic pain, it hurts us to find that we cannot share his uneasiness. If we hear a person loudly lamenting his misfortunes, which, however, upon bringing the case home to ourselves, we feel, can produce no such violent effect upon us, we are shocked at his grief; and, because we cannot enter into it, call it pusillanimity and weakness. It gives us the spleen, on the other hand, to see another too happy or too much elevated, as we call it, with any little piece of good fortune. We are disobliged even with his joy; and, because we cannot go along with it, call it levity and folly. We are even put out of humour if our companion laughs louder or longer at a joke than we think it deserves; that is, than we feel that we ourselves could laugh at it. (Smith, 1759)

Therefore, holding all else constant, any individual will be most pleased when the emotions felt by the spectator matches the emotions he experiences. If the spectator experiences a utility level different from his own, then holding other sources of utility constant, the person principally concerned will be less satisfied than is otherwise possible. This ability or inability of an individual to “enter into,” or empathize with, the actions and emotions of another in a given situation is what Adam Smith referred to as approbation or disapprobation.

Another concept discussed in *Moral Sentiments* is the role of the impartial spectator. Throughout Adam Smith's work, the impartial spectator takes on a variety of identities including literal spectators, society, one's conscience, and a deity. While the behavior and judgments made by these different identities differ in some ways, they by and large act the same, will typically come to the same conclusions, and have nearly identical impact on the utility resulting from an individual's course of action. Therefore, the model of utility discussed in this paper will consider only one kind of impartial spectator that behaves in ways characteristic of the impartial spectator throughout Smith's discourse. The degree of influence that societal standards, internal conscience, religious beliefs, and social affiliations have on one's behavior differs significantly from person to person and would require an extensive amount of theoretical and empirical research that is largely beyond the scope of the topic at hand.

That being said, Smith's general conception of the impartial spectator proves to be remarkably useful in giving an account for altruistic behavior observed in both experimental and day-to-day settings. The impartial spectator plays the role of the great arbiter of justice in Adam Smith's philosophy. As in the case of the invisible hand, it is a natural force that causes individuals to behave in a fashion that is beneficial to the whole, though individuals obey it only out of self-interest. The impartial spectator is the source of the satisfaction one feels after doing what is admirable, and the source of remorse after doing what is contemptible.

One individual must never prefer himself so much even to any other individual, as to hurt or injure that other, in order to benefit himself, though the benefit to the one should be much greater than the hurt or injury to the other. The poor man must neither defraud nor steal from the rich, though the acquisition might be much more beneficial to the one than the loss could be hurtful to the other. The man within immediately calls to him, in this

case too, that he is no better than his neighbor, and that by this unjust preference he renders himself the proper object of contempt and indignation of mankind; as well as of the punishment which that contempt and indignation must naturally dispose them to inflict, for having thus violated one of these sacred rules, upon the tolerable observation of which depend the whole security and peace for human society. There is no commonly honest man who does not more dread the inward disgrace of such an action, the indelible stain which it would for ever stamp upon his own mind, than the greatest external calamity which, without any fault of his own, could possibly befall him; and who does not inwardly feel the truth of that great stoical maxim, that for one man to deprive another unjustly of any thing, or unjustly to promote his own advantage by the loss or disadvantage of another, is more contrary to nature, than death, than poverty, or in his external circumstances. (Smith, 1759)

The individual is interested in the approval of the impartial spectator because the degree to which the impartial spectator approves or disapproves of the actor's actions has a large degree of influence on the actor's level of utility corresponding to each course of action available to him. Therefore, each individual will consider not only the utility of himself and those affected by his actions, but will also consider the justice with which he treats those around him.

The final concept from *Moral Sentiments* that will play a key role in the presented analysis is the distinction between an impartial spectator and a corrupt spectator. An impartial spectator evaluates a situation in its entirety and forms his judgments based on this perspective, while a corrupt spectator will choose to focus only on the aspects of the circumstances that are pleasing and will ignore or disapprove of alternative perspectives on grounds that they are less fanciful as opposed to less objective. It is the nature of a corrupt spectator to focus

exclusively on the individual who is better off and ignoring those who may have been wronged.

The rich man glories in his riches, because he feels that they naturally draw upon him the attention of the world, and that mankind are disposed to go along with him in all those agreeable emotions with which the advantages of his situation so readily inspire him. At the thought of this, his heart seems to swell and dilate within him, and he is fonder of his wealth, upon that account, than for all the other advantages it procures him. The poor man, on the contrary, is ashamed of his poverty. He feels that it either places him out of the sight of mankind or, that if they take any notice of him, they have, however, scarce any fellow-feeling with the misery and distress which he suffers. He is mortified upon both accounts. For though to be overlooked, and to be disapproved of, are things entirely different, yet as obscurity covers us from the daylight of honor and approbation, to feel that we are taken no notice of, necessarily damps the most agreeable hope, and disappoints the most ardent desire, of human nature. The poor man goes out and come in unheeded, and which in the midst of a crowd is in the same obscurity as if shut up in his hovel. Those humble cares and painful attentions which occupy those in his situation, afford no amusement to the dissipated and the gay. They turn away their eyes from him, or if the extremity of his distress forces them to look at him, it is only to spurn so disagreeable an object from among them. The fortunate and proud wonder at the insolence of human wretchedness, that it should dare to present itself before them, and with the loathsome aspect of its misery presume to disturb the serenity of their happiness. The man of rank and distinction, on the contrary, is observed by all the

world. Everybody is eager to look at him, and to conceive, at least by sympathy [empathy], that joy and exultation with which his circumstances naturally inspire him. (Smith, 1759)

Because these corrupt individuals are observing others with whom they have no particular connection, they behave in a similar manner as the impartial spectator. The key exception is that the corrupt spectator is altogether indifferent to any injustice or wrongdoing that may take place. While this concept of the corrupt spectator will arise in the analysis of the ultimatum game, it will generally be assumed that the spectator is impartial and truly concerned about the justice of a given situation.

IV. The Model

The objective of this section is to develop a model that gives an account of rational, utility-maximizing individuals acting in ways that are often assumed to result from altruistic motives. The model will borrow concepts from game theory, general equilibrium, and concepts from *Moral Sentiments* as formerly discussed.

DEFINITION 4.1 Allocation: An allocation is an n -tuple $X = (x_1, x_2, x_3, \dots, x_N)$ such that the x_i is the proportion of a commodity allocated to Player i . All allocations must satisfy $x_i \geq 0, \forall i \in I$ and $\sum_{i=1}^N x_i = 1$.

A population is defined as a set $I = \{1, 2, 3, \dots, N\}$ of N individuals $i \in I$. There is an assumed spectator not in the population that will be referred to as individual 0. All individuals in the population have two utility values corresponding to any feasible outcome or allocation of *Commodity X*.

The first kind of utility is immediate utility and assumes the characteristics traditionally attributed to the utility functions in economics.

DEFINITION 4.2 Immediate Utility Function:
 An immediate utility function of Player i is a function

$$U_i : [0, 1] \rightarrow \mathbb{R}$$

mapping the proportion to a real number denoting Player i 's immediate utility corresponding to that particular proportion. All immediate utility functions must possess the following properties $\forall i \in I$:

1. $U_i(\lambda x_{i,1} + (1-\lambda)x_{i,2}) > \lambda U_i(x_{i,1}) + (1-\lambda)U_i(x_{i,2});$
 $\forall x_{i,1}, x_{i,2}, \lambda \in [0, 1]$ (strict concavity)
2. $x_{i,2} \geq x_{i,1} \implies U_i(x_{i,2}) \geq U_i(x_{i,1});$
 $\forall x_{i,1}, x_{i,2} \in [0, 1]$ (increasing in x)

Immediate utility functions display diminishing marginal utility (implied by strict concavity) while accommodating the generally accepted assumption that all else equal, individuals will prefer to have more of a good than less. This model assumes that all individuals place the same level of value on any given quantity or proportion of *Commodity X* meaning that every individual's immediate utility functions are identical.

While immediate utility functions are typically sufficient in explaining the behavior of individuals in an economic context, one area that their predictions have proved unsatisfactory is in the context of various game theoretic situations, particularly those of the dictator game, ultimatum game, and public goods games. This paper focuses on the first and second of these three settings.

As discussed extensively in the literature review, the behavior of individuals involved in experimental dictator and ultimatum games behave in a fashion that does not appear to maximize their own utility defined above. The most apparent reason for this deviation from the assumed characteristics of human nature characteristic of economic theory, particularly those of self-interest and rationality, is the failure for traditional utility to take into account the tendency of humans to

empathize with those around them, ironically a point discussed by the Adam Smith before he ever proposed his analysis and defense of the free market which has become the foundation of economic theory. This model will take into account this tendency of individuals to empathize with the experiences of others.

DEFINITION 4.3 Empathy Function: An Empathy Function

$$\Gamma_{i,j}(x_i, x_j; \gamma_{i,j}) : [0, 1] \times [0, 1] \rightarrow [0, 1]$$

is defined as a function mapping the proportions of *Commodity X* allocated to two individuals $i, j \in I$ to a number denoting the amount of utility Player i derives in proportion to the immediate utility experienced player j . Player i 's empathy function corresponding to Player j is of the form:

$$\Gamma_{i,j}(x_i, x_j; \gamma_{i,j}) = \begin{cases} \left(\frac{1}{\Gamma_{i,j}^r}\right)^p \left(\frac{dU_i(x_i)}{dU_j(x_j)}\right)^q \gamma_{i,j}^r & x_i, x_j, \gamma_{i,j} \in [0, 1]; x_i + x_j \leq 1; q, r \in \mathbb{N}; p \in \mathbb{N}_{0,q}^1 \\ 0 & \text{otherwise} \end{cases}$$

where $\sum_{j=1}^N \gamma_{i,j} = 1$ and $\Gamma_{i,j}^* = \frac{dU_i(1)}{dU_j(0)}$.

In order to form a more complete depiction of utility one ought to consider the utility an individual derives by observing the happiness of those around him. But the level of empathy one has for another varies according to two factors. The first is the degree of influence that the happiness of that individual j in particular has on the overall satisfaction of individual i . Mathematically, this takes the form of a scalar $\gamma_{i,j} \in [0, 1]$ that represents the degree of influence that j 's immediate utility has on i 's altruistic utility level where $\sum_{j=0}^N \gamma_{i,j} = 1, \forall i \in I$.²

¹ $\mathbb{N}_{0,q}$ denotes the set $\mathbb{Z} \cap [0, q]$

²Each individual i in the population includes a scalar $\gamma_{i,0}$ corresponding to their sensitivity to the opinion of the impartial spectator, which will be discussed in more detail. In the case of i 's empathy with the impartial spectator, this is the *only* factor that plays a role in determining what influence of the impartial spectator on i 's overall utility.

The second is a consideration for the well being of the observed individual in comparison to that of the observer. In order to do so, one could simply calculate the ratio between the amount of *Commodity X* the other possesses to the amount possessed by himself. While this approach may prove useful, it fails to consider the law of diminishing marginal utility. An alternative is to use the ratio of the marginal utilities of the two players at their respective quantities of *Commodity X* possessed. Assuming diminishing marginal returns and identical immediate utility functions, the individual with a smaller share of *Commodity X* will have a larger marginal utility value. This approach offers several advantages that the former does not. First of all, this means that in increasingly unequal the circumstances, the better-off individual will become increasingly willing to sacrifice more than they would otherwise in order to raise the utility level of his disadvantaged companion. Another consequence of this is that in an N -person context, individuals will prefer to see a worse-off individual gain an additional unit of the commodity being allocated as opposed to a better-off individual, holding all else constant. Finally, common experience also offers a validation of this perspective in that individuals tend to give to those whom are worse of than themselves, and furthermore individuals typically give only what they have in excess.

Now that a formal definition of empathy has been established, the final component that needs to be defined in order to introduce the concept of altruistic utility is the idea of the spectator utility function. Because the spectator is an individual that merely observes the game without possibility of any tangible benefit or harm, he has no immediate utility function and therefore, there cannot be any ratio of marginal immediate utility. Therefore, the impartial spectator's empathy functions for each individual i , $\Gamma_{0,i}$, are simplified to the scalars $\gamma_{0,i}$.

DEFINITION 4.4 Spectator Function: A Spectator Function is a function

$$\Lambda_0(X; \vec{\gamma}_0) : [0, 1]^N \rightarrow [0, 1]$$

mapping the proportions allocated to each of the N players to a number denoting the utility level of the spectator corresponding to the outcome of those moves. Note that the parameters $\gamma_{0,i} \in [0, 1]$ represent the spectator's empathy level with individual i and must satisfy $\sum_{i=1}^N \gamma_{0,i} = 1$. There is only one Spectator Function corresponding to any game.

A crucial aspect of the way that the spectator function has been defined is that the vector $\vec{\gamma}_0$ is an allocation vector where the commodity is the approval of the spectator. This will play a large role in analyzing and predicting the behavior of individuals in game theoretic settings in the following sections.

The final concept to be discussed is the concept of altruistic utility. The purpose of altruistic utility is to provide a more complete understanding and depiction of utility by taking into account one's immediate utility, his empathy with those around him, and his empathy and sensitivity to the utility and opinion of the impartial spectator.

DEFINITION 4.5 Altruistic Utility Function:
 An altruistic utility function of Player i is a $(N+1)$ -
ary function

$$\mathcal{U}_i(X, \gamma_{0,i}; \vec{\gamma}_i) : [0, 1]^N \rightarrow [0, 1]$$

mapping an allocation X of Commodity X and the proportion of approval or empathy $\gamma_{0,i} \in [0, 1]$ the impartial spectator designates to Player i . All altruistic utility functions must satisfy the following properties:

1. $\mathcal{U}_i(X, \gamma_{0,i})$ is continuous over the domain
2. $\mathcal{U}_i(X, \gamma_{0,i})$ is quasiconcave in x_i .

In the following section, a utility function of this kind will be constructed in hopes of providing a plausible explanation of the unexpected behavior displayed in various game theoretic experiments as discussed in the literature.

A. Altruistic Utility in the Dictator Game

The objective of this section is to use the model described in the previous section to give an account for the giving that frequently occurs in experimental dictator games. In the dictator game, Player I (i.e.: the dictator) divides a particular sum of money between himself and another individual. The other individual has no control over the amount either individual will have in the end. Traditional economic theory would predict that the dictator, like everyone, is rational and operates on the principle of self-interest. Hence the dictator will simply give himself the whole sum, leaving nothing for his companion. As mentioned in the literature review, however, when dictator games actually take place in experimental settings, dictators have often given away a considerable, yet relatively small, amount away, while aware that they are under no obligation to do so. Such results seem to suggest that there are ulterior motives at play that may cause individuals to give

away some proportion of the money away at the expense of their immediate utility.

Suppose the dictator has the immediate utility function

$$U_1(x_1) = \frac{\ln(1 + x_1)}{\ln(2)} \quad (1)$$

where $x \in [0, 1]$ denotes the proportion of the total sum of money that the dictator gives to himself and $x_2 = 1 - x_1$ is the proportion allocated to Player II. We define U_1 to be zero for any $x \notin [0, 1]$.

In order to construct a potential altruistic utility function that for Player I, one must next consider his empathy with Player II. Hence, his hypothesized utility function for Player II is the same as his own, except with the payoff $x_2 = 1 - x_1$ taking the place of his own, x_1 . As Adam Smith said in *Moral Sentiments*,

As we have no immediate experience of what other men feel, we can form no idea of the manner in which they are affected, but by conceiving what we ourselves should feel in the like situation.... It is the impressions of our own senses only, not those of his, which our imaginations copy. By the imagination we place ourselves in his situation, we conceive ourselves enduring all the same torments, we enter, as it were into his body, and become in some measure the same person with him, and thence form some idea of his sensations, and even feel something which, though weaker in degree, is not altogether unlike them (pages 1-2).

Because Player I cannot know for certain Player II's valuation for each possible proportion of whatever is currently the commodity of interest, Player I can only assume that Player II's immediate utility function is the same as his own. Because Player II has no influence over the outcome of the dictator game, it can only be assumed that Player II's immediate

utility function is

$$U_2(x_2) = \frac{\ln(1 + x_2)}{\ln(2)}$$

or equivalently

$$U_2(x_1) = \frac{\ln(2 - x_1)}{\ln(2)}. \quad (2)$$

Given these immediate utility functions corresponding to each Player I and Player II respectively, one can form $\Gamma_{1,2}$ as formerly defined, that is, Player I's empathy function corresponding to Player II. The nontrivial portion of the empathy function is:

$$\Gamma_{1,2}(x_1, x_2; \gamma_{1,2}, p, q, r) = \left(\frac{1}{2}\right)^p \left(\frac{1 + x_2}{1 + x_1}\right)^q \gamma_{1,2}^r.$$

For convenience, this can be written in an equivalent form that is a function of x_1 only:

$$\Gamma_{1,2}(x_1; \gamma_{1,2}, p, q, r) = \left(\frac{1}{2}\right)^p \left(\left|\frac{x_1 - 2}{1 + x_1}\right|\right)^q \gamma_{1,2}^r = \left(\frac{1}{2}\right)^p \left(\frac{2 - x_1}{1 + x_1}\right)^q \gamma_{1,2}^r. \quad (3)$$

For the sake of simplicity, subsequent functions corresponding to Player II will be expressed as functions of x_1 by substituting $x_2 = (1 - x_1)$ as shown above.

The next key component in Player I's altruistic utility function is the spectator function. Because the spectator has no personal stake in the outcome of the observed situation, he is far more interested in the overall justice of the outcome than the actual utility of the individual players. Therefore, the spectator function will have an empathic function corresponding to each player, but with a far higher degree of sensitivity towards the marginal rate of substitution of the immediate utility of the different players and less on the actual level of utility experienced by those individuals. Hence, in the model presented, the spectator function will be defined as

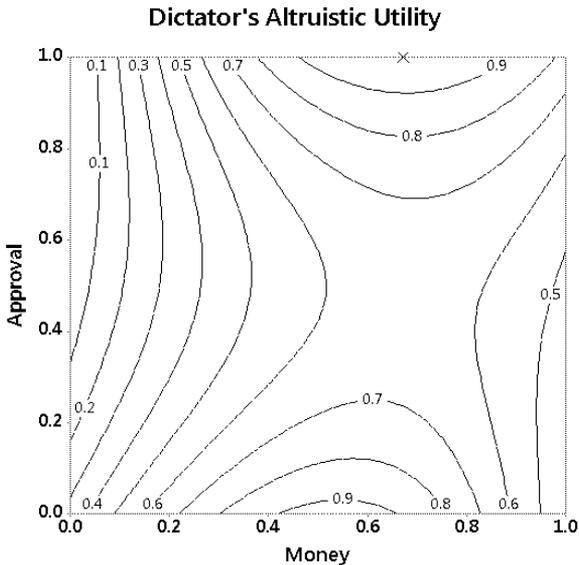
$$\Lambda_0 = \frac{1}{0.4347} \left[\left(\frac{\ln(1+x_1)}{\ln(2)} \right)^2 \left(\frac{2-x_1}{1+x_1} \right)^2 \gamma_{0,1}^3 + \left(\frac{\ln(2-x_1)}{\ln(2)} \right)^2 \left(\frac{1+x_1}{2-x_1} \right)^2 (1-\gamma_{0,1})^3 + 0.5 \left(\frac{\ln(1+x_1)}{\ln(2)} + \frac{\ln(2-x_1)}{\ln(2)} \right) - 0.5 \right] \quad (4)$$

The above spectator function takes the form of a three-dimensional saddle whose contours are displayed in Figure 1 below.

Finally, an altruistic utility function for Player I can be constructed by taking a linear combination of these three components (Player I's immediate utility function, the product of Player I's empathy with Player II and Player II's immediate utility, and the spectator function) and adjusting the location and slopes in order to satisfy the restrictions laid out in the definition of altruistic utility functions.

$$\mathcal{U}_1(x_1, \gamma_{0,1}) = (1.745) \left[\left(\frac{1}{3} \right) \frac{\ln(1+x_1)}{\ln(2)} + \left(\frac{1}{3} \right) \left(\frac{\frac{2-x_1}{1+x_1}}{2} \right) \frac{\ln(2-x_1)}{\ln(2)} + \left(\frac{1}{3} \right) \Lambda_0(x_1, \gamma_{0,1}) - 0.084 \right]. \quad (5)$$

The graph of the altruistic utility function takes the form of a saddle mapping each combination of approval and money to the dictator's corresponding value of altruistic utility. The contour graph of this surface is displayed below.



Because this is a utility function with two “commodities,” money and the spectator’s approval, one can produce indifference curves for the dictator. In the case of the dictator game, the dictator strives to maximize his own altruistic utility while also trying to satisfy his own conscience (i.e.: the impartial spectator). Therefore, in order to determine the split that will be chosen by the dictator, we need only to solve for the value of x that corresponds to the highest contour of the dictator’s altruistic utility function. Because there is no conflict between players, as there will be in the case of the ultimatum game, the impartial spectator plays a passive role and merely maximizes his own utility in response to how the dictator behaves. Hence, given the altruistic utility function just presented, the dictator will choose to allocate approximately 0.6695 of the money to himself and give the remaining 0.3305 the other individual. This allocation of money corresponds to the “ \times ” symbol on the diagram above marking the maximum point of U_1 over the domain $\mathcal{D} = [0, 1] \times [0, 1]$. While this result is a considerably more generous division on the part of

the dictator than what is encountered in the empirical literature, it does seem to suggest that such a utility function as described and constructed in this fashion may prove useful in providing an account for the altruistic behavior in the dictator game. Furthermore, one may acquire more accurate results by employing either maximum likelihood estimation or nonparametric techniques in order to derive more reliable values for the parameters $\gamma_{i,j}$ of the spectator and altruistic utility functions.

B. Altruistic Utility in the Ultimatum Game

The ultimatum game is similar to that of the dictator game, except that Player II has a say in the final outcome of the game. In the ultimatum game, Player I proposes a split as before, but Player II then decides whether or not he accepts the proposed split. If the split is accepted, each individual receives the amount specified by Player I. If the proposed split is rejected however, neither player receives any money, leaving each player in the same state as they were before the game began.

In this model, it is assumed that communication is permitted between the two individuals and that both players move simultaneously. As was the case in the dictator game, Player I's move x_1 is the proportion that Player I allocates to himself, leaving $(1 - x_1)$ for Player II. Player II's move x_2 is the proportion below which Player II will reject the split. In other words, x_2 is the minimum proportion that Player II will accept. Therefore, the split will be rejected and both players will receive a payoff of zero if and only if $(1 - x_1) < x_2$. For the sake of simplicity, it is assumed that Player I's altruistic utility function and the spectator utility function are the same as was in the case of the dictator game. For now it will be assumed that Player II has the same utility assignments for each feasible combination of money and approval. In order to find potential equilibrium points one must first remember that each individual cares first and foremost of maximizing their altruistic utility, and cares about their companion's well-being only in so far as it enables each of them to achieve their respec-

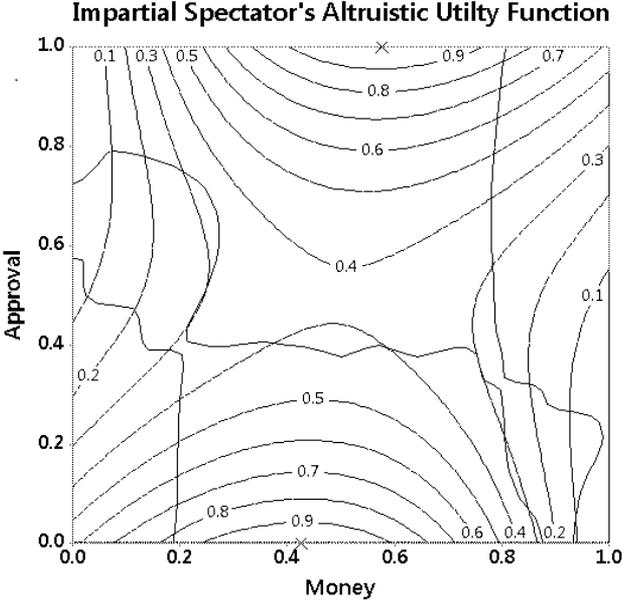
tive ends. Clearly, no individual wants to reach an outcome in which the proposed split is rejected. However, as has been demonstrated by the behavior of the above function, as well as by Smith's intuition, while each individual naturally shows some concern for the other's well being, each is far more deeply concerned and affected by his own. Therefore, each player's utility-maximizing split gives a considerably higher proportion to themselves, though not all of it due to the influence of the impartial spectator and their own empathy for the other. This was most clearly demonstrated in the analysis of the dictator game in the former section. However, due to the different circumstances characteristic of the ultimatum game, Player I must now consider far more seriously the preferences of Player II due to the substantially larger influence Player II's preferences now have on his own utility. Therefore, the primary opponent of Player I is no longer his own conscience, but the individual with whom he shares the money. Provided that each player does not receive so large a proportion that his altruistic utility begins to fall as x rises (a proportion so large that not even a dictator would choose), each individual prefers a larger proportion for himself than a smaller one. Hence, Player I wants to offer no more than the minimum amount Player II will accept and likewise, Player II want to ask for no less than the maximum amount that Player I is willing to sacrifice. The contract curve $\mathcal{C}_{1,2}$ shows not the proportions that each individual is able to justify to himself (i.e.: his conscience), but rather those that he is able to justify to the other. Therefore, the contract curve defined below presents the set of potential splits between Players I and II.

$$\mathcal{C}_{1,2} \equiv \frac{\frac{dU_1}{dx}}{\frac{dU_1}{d\gamma_0}} = \frac{\frac{dU_2}{dx}}{\frac{dU_2}{d\gamma_0}} \quad (6)$$

The way the observed allocation in particular is chosen however, is the role of the impartial spectator. This is observed in the face of conflict in nearly every sphere of social and political life. The way individuals argue among what realistic courses of action ought to be taken is through appeals

to justice, morals, ethics, or fairness. These judgments are reserved for the impartial spectator. As previously established, the spectator utility function is higher at allocations that more nearly reflect and satisfy the ideals of justice. Therefore, the allocation along the contract curve $\mathcal{C}_{1,2}$ that is actually observed is the one that corresponds to the highest point on the spectator function. Mathematically, this takes the form of an optimization problem with Λ_0 as an objective function over the domain $\mathcal{D} = [0, 1] \times [0, 1]$ subject to constraint function $\mathcal{C}_{1,2} \subset \mathcal{D}$ jointly created by Players I and II. In addition to the contract curve there is one additional constraint that often and currently applicable. Because the spectator now plays an active role in determining the equilibrium of the game, one must determine whether the spectator is truly impartial or is corrupted in the manner described by Smith in *Moral Sentiments*. Smith said that a corrupt individual would empathize not with those whom are worse off, but actually be indifferent to them and focus their attention and sympathies on the elites of their society, in order to derive pleasure by empathizing with the pleasure of the wealthy and powerful. In this manner, one must determine if the spectator in the context of the ultimatum game is truly impartial or corrupt. Recall that though the spectator takes on many forms it, by and large, behaves the same way and comes to the same conclusions. Thus, since the actual spectators of the ultimatum and dictator games, have demonstrated a pattern of attributing positive terms to the unexpectedly high shares given away by subjects, such as altruism or cooperation, as opposed to negative ones such as arrogance or manipulation, one may conclude that the spectator is truly impartial and does not empathize disproportionately in favor of the one who gains a higher share of money in the end. For this reason it is assumed that when there is more than one possible level of approval for a given split of money, the impartial spectator will give the maximum amount of approval or empathy to the individual with smaller share of money, subject to the constraints of the contract curve. The corresponding graph for finding the resulting allocation

of the ultimatum game is displayed below where the contours are those of the impartial spectator's altruistic utility function and the three remaining curves are the three portions of the contract curve derived from the altruistic utility functions of Players I and II.



Note that due to the impartiality of the spectator, the portion of the far-right contract curve is only relevant when the x_1 is greater than the x -intercept value of the middle portion of the contract curve. Similarly, the portion of the contract curve in the southwest region of the graph is also inapplicable, as the choice of an allocation along that portion of the contract curve would require the spectator to be nearly indifferent to the injustice associated with such a disproportionate allocation between the two individuals. Therefore, the resulting amount of money given to Player I will be about 0.5699, leaving about 0.4301 for Player II. While these results are slightly more accurate than the empirical outcomes, statistical approximations of the parameters involved such as those previously mentioned would provide more meaningful predictions, as well as a more

useful evaluation of the model at hand.

V. Conclusion

This paper has presented a model that demonstrates rational players acting upon the principle of self-interest giving away significantly more money than is necessary. The philosophical groundwork of the model, provided primarily by Adam Smith, proves instrumental not only in determining how rational individuals will behave in various strategic and non-strategic circumstances, but also in grasping all that is implied under the term “self interest.” Smith makes it clear that while humans are naturally empathetic, they may try to direct their attention in a manner so that this natural response will bring them only empathetic pleasure and not empathetic pain. For this reason, it seems reasonable that this natural tendency is not so much a contradiction to the principle of self-interest, but a complication of it. Therefore, before deeming certain actions as altruistic and attributing to them the appropriate approbation, one should determine if the action was done in order to receive this approbation or done in spite of the forthcoming approbation.

While this model demonstrates much potential in accounting for the regard individuals have for one another, as well as their regard for the societal standards for proper behavior, there are some flaws that indicate room for improvement. The most obvious of these is the degree to which the model appears to overestimate the level of cooperation and empathy demonstrated in their behavior. The model predicts far more equal shares in favor of Player II than what is seen in the empirical literature. Perhaps the largest source of this error is the choice of the values for the parameters $\gamma_{1,1}$, $\gamma_{1,2}$, and $\gamma_{1,0}$. The results of the model would be far more accurate if these were estimated with maximum likelihood estimation or some other empirical means, and then new bounding parameters were calculated to ensure the output of the modified model satisfied the required properties laid out in the definition of altruistic utility.

Another concept that, if successfully integrated into the model, would further increase the insight provided by the model is that of uncertainty regarding the preferences of others and their degree of sensitivity to rules of fairness and your own immediate utility, which translate to uncertainty regarding the likelihood of a positive payoff in the dictator game. An analysis into potential causes of a spectator remaining impartial or becoming corrupt may also provide a deeper understanding of the formation and evolution of public opinion in a political and legal context. Such an analysis would further raise a number of provocative questions on how one may remedy such problem and the ethics revolving around such deliberations.

Finally, one may object that the term “altruism” has been used improperly since players receive intrinsic rewards for splitting the sum more fairly and hence, are done simply to maximize one’s modified utility function. This paper uses the term “altruistic” to describe an act whose *immediate* costs and benefits sum to an economic loss, resulting in the appearance of a contradiction of the *homo economicus* assumption of economic theory. In that case, this paper is discussing not what altruism is, but what altruism is not. Adam Smith’s philosophy is not being used to explain altruism, but to better define it.

References

- [1] Abbott, Stephen. 2001. *Understanding Analysis*. New York: Springer.
- [2] Ashraf, Nava, Colin F. Camerer, and George Loewenstein. "Adam Smith, Behavioral Economist." *Journal Of Economic Perspectives* 19, no. 3 (Summer 2005): 131-145. Business Source Premier, EBSCOhost (accessed October 16, 2015).
- [3] Barron, E.N. 2013. *Game Theory: An Introduction*. New Jersey.
- [4] Bohnet, Iris, and Bruno S. Frey[*]. 1999. "Social Distance and Other-Regarding Behavior in Dictator Games: Comment." *American Economic Review* 89, no. 1: 335- 339. Business Source Premier, EBSCOhost (accessed October 16, 2015).
- [5] Burks, Derek J., Lorraine K. Youll, and Jayson P. Durtschi. "The Empathy-Altruism Association and It Relevance to Health Care Professionals." *Social Behavior and Personality: An International Journal* 40, no. 3 (April 2012): 395-400. SPORTDiscus, EBSCOhost (accessed October 16, 2015).
- [6] Cox, James C., Daniel Friedman, and Steven Gjerstad. "A Tractable Model of Reciprocity and Fairness." *Games & Economic Behavior* 59, no. 1 (April 2007): 17-45. Academic Search Complete, EBSCOhost (accessed December 16, 2016).
- [7] Eckel, Catherine, et al. "Social Norms of Sharing in High School: Teen Giving in the Dictator Game." *Journal of Economic Behavior & Organization* 80, no. 3 (December 2011): 603-612. Academic Search Complete, EBSCOhost (accessed December 16, 2016).
- [8] Eugene Heath. "The Commerce of Sympathy: Adam Smith on the Emergence of Morals." *Journal of the*

- History of Philosophy* 33, no. 3 (1995): 447-466.
<https://muse.jhu.edu/> (accessed October 17, 2015).
- [9] Evensky, Jerry. "Adam Smith's Theory of Moral Sentiments: On Morals and Why They Matter to a Liberal Society of Free People and Free Markets." *Journal Of Economic Perspectives* 19, no. 3 (Summer2005 2005): 109-130. Business Source Premier, EBSCOhost (accessed October 16, 2015).
- [10] Henrich, Joseph, et al. "Economic Man in Cross-Cultural Perspective: Behavioral Experiments in 15 Small-Scale Societies." *Behavioral & Brain Sciences* 28, no. 6 (December 2005): 795-855. Academic Search Complete, EBSCOhost (accessed December 16, 2016).
- [11] Hoffman, Elizabeth, Kevin McCabe, and Vernon L. Smith. "Social Distance and Other-Regarding Behavior in Dictator Games." *The American Economic Review* 86, no. 3 (1996): 653-660. <http://www.jstor.org/stable/2118218>.
- [12] Koch, Alexander K. and Norman, Hans-Theo. "Giving in Dictator Games: Regard for Others or Regard by Others?" (August 2005). IZA Discussion Paper No. 1703. Available at SSRN <https://ssrn.com/abstract=779067>.
- [13] Levin, Jonathan. General Equilibrium. 2006. TS, November. Accessed Fall 2016. <https://web.stanford.edu/~jdlevin/Econ%20202/General%20Equilibrium.pdf>.
- [14] Louis Kaplow and Steven Shavell. "Moral Rules, Moral Sentiments, and Behavior: Toward a Theory of Optimal Moral System." 115 J Pol. Econ. 494 (2007).
- [15] Podkopaev, Dmitry. "An Approach to Modeling Altruistic Equilibrium in Games." International Workshop On Multiple Criteria Decision Making (January 2010): 188-199. Academic Search Elite, EBSCOhost (accessed October 16, 2015).

- [16] Oosterbeek, Sloof, and Van de Kuilen. “Cultural Differences in Ultimatum Game Experiments: Evidence from a Meta-Analysis.” *Experimental Economics* no. 7 (June 2004): 171-188. doi:10.1023/B:EXEC.0000026978.14316.74
- [17] “Open Yale Courses.” Open Yale Courses. Accessed December 16, 2016. <http://oyc.yale.edu/>.
- [18] Smith, Adam. 1759. *The Theory of Moral Sentiments*. New York: Cosmo Classics.

Estimating Minority Benefit from Government Assistance in the US Mortgage Market

Maksim Papenkov[†]

Abstract

This study reviews a series of federal programs developed to alleviate difficulties faced by lower-income and minority applicants in the mortgage origination process. I use HMDA LAR data to measure the historical origination gap between various minority groups, and identify the relative benefit that each group receives upon utilizing government assistance, through programs such as FHA and VA. Previous literature has identified that certain minority groups tend to have more difficulty in the origination process, by having higher interest rates on their loans, and a higher likelihood of denial in the application process. This paper builds on this discussion by identifying trends in mortgage origination on a national level, over the past 27 years. This study identifies that historically, Black applicants have been more likely to utilize government assistance programs than White or Asian applicants, across all income levels. Though Black applicants benefit the most while utilizing these programs, the Black-White origination gap has not significantly decreased over the last three decades, indicating that the underlying causes of this disparity remain unaffected. Additionally, this study identifies that same-sex applicant pairs have lower origination rates relative to their single and male-female pair counterparts, indicating potential discrimination on the basis of sexual orientation, by the mortgage lender.

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I. Introduction

Over the past eighty years, the prospect of homeownership in the United States has transitioned from a personal ambition to a national imperative. A series of federal programs have developed to alleviate difficulties faced by lower-income applicants while acquiring credit, and a group of civil rights laws mitigate active discrimination against minorities in the mortgage market. Despite these actions, the homeownership gap between White and Black Americans was approximately 30.5% in 2016, while the origination¹ gap was approximately 12.6% in the same year (FFIEC, 2018; United States Census, 2017). Though an abundance of previous literature has attributed this disparity to economic factors correlating with applicant race - including wealth and debt history - recent housing data indicates that this disparity has not substantially diminished over the past two decades, demonstrating that although minority applicants have been able to obtain assistance through federal programs, disadvantage still permeates the mortgage market. This paper aims to estimate which minority groups are most likely to utilize these programs, and whether their likelihood of origination is higher relative to their conventional loan counterparts.

Two barriers hinder lower-income applicants in the origination process. The first is a moral hazard problem faced by lenders, in which they have an incentive to deny credit to applicants with a moderate risk of default; the second is the difficulty for an applicant to obtain the sufficient funds necessary for a prerequisite down payment, which was an average of 11% of the total loan value in 2016 (DeSanctis, 2017). Though no federal programs directly provide a line of credit in the form of a mortgage, two instruments have been developed to assist applicants in obtaining a loan through a private lender: federal loan insurance and federal loan guarantees. Loan insurance and loan guarantees operate similarly, though insurance

¹Origination involves a mortgage that is both: (1) approved by the lender and (2) accepted by the borrower.

requires a monthly premium, while a guarantee does not. By assuming an applicant's debt obligation in the case of a default, the federal government - backed by the U.S. Treasury - eliminates the lender's moral hazard problem, allowing qualifying applicants with lower credit scores to obtain a mortgage. Further, each of these programs address the second barrier by either allowing for an origination with no down payment, or by setting a cap on the required percentage.

Four such federal programs are currently in operation. The first is the National Housing Act of 1934, passed as a part of FDR's New Deal during the peak of the Great Depression. It established the Federal Housing Administration (FHA), an organization designed to revitalize the U.S. housing market during a period of rapidly declining home ownership, due to a decrease in mortgage originations and a rise in foreclosures. The FHA provides federal loan insurance to qualifying applicants, with a 3.5 percent limit on down payments, significantly lower than what is generally available to conventional borrowers. Separately, the Serviceman's Readjustment Act of 1944 (otherwise known as the "G.I. Bill") established the Veteran Affairs (VA) Loan Program, which provides loan guarantees to qualifying veterans with no stipulation regarding an applicant's credit history or debt-to-loan ratio; the VA Loan Program does not require the applicant to provide a down payment (United States Department of Veterans Affairs, 2018). Additionally, the Farm Service Agency (FSA) and the Rural Housing Service (RHS) provide mortgage assistance and loan guarantees to low-income individuals living in rural areas (Farm Service Agency, 2012; Rural Development, 2017). Despite the success of such programs in assisting some applicants in the mortgage origination process, not all minority groups have historically been able to take advantage of these federal programs with comparable success. Specifically, a paper by Hanchett (2000) finds that "Between 1945 and 1959, less than 2 percent of all federally insured home loans went to African Americans."

Several federal laws have attempted to manage discrimi-

natory mortgage lending practices directly. The Fair Housing Act, passed under Title VIII of the Civil Rights Act of 1968, made it illegal to refuse to sell or rent a home based on sociodemographic characteristics, including race and ethnicity, which was later expanded to further include protection based on applicant sex in 1974. Soon after, the Equal Credit Opportunity Act of 1974 made it illegal for lending institutions to deny credit based on those characteristics, and in the case of a mortgage denial, they were required to disclose a specific reason to the applicant. Finally, The Community Reinvestment Act was passed by Congress in 1977 to directly prohibit redlining, the process of systematically denying credit to applicants living in minority urban neighborhoods. Despite the enforcement of these laws for several decades, racial disparities in mortgage lending still persist.

In an attempt to be able to accurately measure these disparities in the mortgage market, the Home Mortgage Disclosure Act (HMDA) was passed by Congress in 1975, and enacted by the Federal Reserve Board through Regulation C, with the purpose of providing a public archive of mortgage application records used to monitor for discriminatory lending patterns. Specifically, HMDA mandates that certain financial institutions disclose summaries of each of their individual mortgage applications to the Federal Financial Institutions Examination Council (FFIEC), which publishes an annual Loan Application Register (LAR) containing information on each individual mortgage processed within a given year. Initially, the data collected by the FFIEC was limited to loan information, though following an amendment to Regulation C in 1989, which incorporates elements of the Financial Institutions Reform, Recovery, and Enforcement Act (FIRREA), the LAR was expanded to include descriptive applicant information, including race and income. To avoid violating individuals' privacy, any identifying information - such as name and address - are omitted from the public data set. HMDA is strictly a measure to collect and publish mortgage application data, and does not establish any specific quotas or restrictions modifying

how financial institutions are required to operate their lending practices.

The first significant paper utilizing the expanded HMDA LAR data set was published by Munnell, Browne, McEneaney, & Tootell (1996), through the Federal Reserve Bank of Boston, which compared mortgage discrimination between White and Black / Hispanic applicants in the Boston Metropolitan Statistical Area (MSA) in 1990.² An observation had been made that between 1990 and 1993, high-income minority applicants were more likely to be denied for a mortgage than low-income White applicants in the Boston area, prompting the question of whether this was due to racial discrimination, or some other underlying factors. To supplement the HMDA LAR data set, the Boston Fed requested information on 38 additional variables from mortgage-lending financial institutions, including applicant age, total value of liquid assets, and total liabilities, among others. The survey sample was limited to all conventional mortgages made by Black / Hispanic applicants and a random sample of 3,300 mortgages made by White applicants in the Boston MSA in 1990. The scope of the study was constrained to a single MSA in a single year due to the costly and labor-intensive process of physically retrieving the mortgage documentation and reporting it back to the Federal Reserve Bank of Boston in a timely manner. The study found that Black / Hispanic applicants were, in fact, more likely to be denied a mortgage relative to White applicants, however it remained unclear as to whether this was due to direct racial discrimination, or economic variables which are highly correlated with applicant race. The researchers argued that their analysis had abstracted from discrimination in other parts of the economy, and that an applicant's difficulty in originating a mortgage was a direct corollary of them facing discrimination in education, as well as in the job market. Black / His-

²The HMDA LAR did not include a parameter for Ethnicity (Hispanic or Latino) until the FIRREA expansion in 2004, so the researchers were not able to control for this directly, but the assumption was made that Hispanic applicants would describe themselves as Black, given the limited choice in possible applicant races.

panic applicants were found to have less wealth and more debt than White applicants, on average, and several other social factors were discussed which may contribute to the discrepancy between mortgage denial rates by applicant race. Though the study was not able to directly identify discrimination in the mortgage market, the research was able to identify that Black/Hispanic applicants were at a disadvantage relative to White applicants, as demonstrated by the differences in denial rates by race.

Several other studies rely on HMDA LAR data to study patterns of discrimination. A study by Robinson (2002) uses the expanded 1990 Boston data to study gender and familial status discrimination, to find that within their sample, single women with children are disadvantaged relative to single men with children, regardless of race, though for male-female couples in which the wife was in the workforce, there was a disadvantage for White couples, while no similar disadvantage was observed for Black/Hispanic couples. Further, a study by Faber (2013) uses HMDA LAR data to study racial differences in the subprime mortgage market to find that Black and Hispanic applicants were more likely to receive a subprime mortgage than White applicants, while Asian applicants were far less likely. When expensive (jumbo) loans were removed from their model, a positive correlation was found between income and the likelihood of obtaining a subprime mortgage for minority applicants, indicating that wealthier minorities were targeted for subprime loans despite being eligible and fully qualified for prime loans. Separately, a study by Smith & Hevener (2014) uses HMDA LAR data to find that credit scores and denial rate for prime conventional loans tend to account significantly for the racial disparity in subprime rates.

Additionally, a study by Lindsey-Taliefero (2015) uses HMDA LAR data from 2004 to 2013 to study gender differences in the mortgage market, to find that on a national level, gender is not a statistically significant determine factor of origination following the housing crisis, though when controlled for race, some significant differences were observed, particu-

larly for White and Asian women. Further, the analysis found that minority women had a more negative mortgage experience than White males, except for 33 percent of the time in which no statistically significant differences were observed. Finally, the study concludes that applicant race has a greater effect on origination than applicant sex. A study by Wheeler & Olson (2015) uses HMDA LAR data from 1990 to 2013 to estimate the relationship between annual metropolitan area-level house price inflation and the White-Black origination gap to find that as house prices increase more rapidly, this disparity decreases, implying that local housing market conditions are significant in explaining some racial disparities in mortgage origination.

Other studies analyze discrimination in the U.S. mortgage market using alternative data sources. A study by Cheng, Lin, & Lin (2015) uses national data from the U.S. Survey of Consumer Finance in 2001, 2004, and 2007 to study differences loan rates by race to find that Black borrowers pay an average of 29 basis points more than white borrowers for their mortgage, particularly younger borrowers with lower income and less education. Specifically, they find that financially vulnerable Black women face the greatest disadvantage in the mortgage market. Another study, by Kau, Keenan, & Munneke (2012) uses a data sample consisting of 2,529 thirty-year fixed-rate conventional loans originated in Miami, Florida between 1975 and 2002 to study racial differences in contract rates to find that borrowers in predominantly Black neighborhoods generally tend to have higher rates than other racial groups. A study by Hilber & Liu (2008) used data from the Panel Study of Income Dynamics (PSID) to find that the White-Black homeownership gap observed in their sample could be entirely explained by using a regression with the traditional explanatory variables (debt, income, etc.) while also including differences in wealth and preferred location type (degree of urbanization). A separate study using PSID data by Charles & Hurst (2002) finds that in addition to having lower origination rates, Black families have a lower propensity to apply, being discouraged by two primary factors. The first is

an anticipated differential treatment by the lender, and the second being a lower tendency to receive financial assistance from their families. Specifically, they find that, within their sample, 27 percent of White households relied on their families to help pay for their down payment, while only seven percent of Black households were able to do so. The researchers conclude that differences in applicant parent wealth explains a portion of the racial disparity in origination rates. A study by Segal and Sullivan (1998) uses March Current Population Survey data between 1977 and 1997 to find that about 40 percent of the White-Black homeownership gap can be explained by both income and demographic factors. Further, a study by Agarwal, Li, & Mielnicki (2003) used a bank-specific approach on 567 loans in 1999 to find that when controlling for an expansive list of economic variables, the results indicate applicant race to be statistically insignificant as an explanatory variable for mortgage origination - indicating that financial and credit-worthiness are the most significant determinants of origination, within their sample data. For a comprehensive review of older literature on the subject, refer to LaCour-Little (1999).

While much of the discussion in the previous literature has been concerned with determining the factors contributing to the disparity in mortgage origination rates between White applicants and various minority groups, little attention has been given to the government assistance programs available to minority applicants. This paper aims to address this gap in the literature, by estimating the relative benefit that each group receives when utilizing such programs. The purpose is to evaluate whether minority mortgage lending difficulties have diminished over the past three decades, and to identify whether government assistance has had any positive effect on origination.

II. Methods

This study utilizes historical HMDA LAR data sets from 1990 to 2016, which were retrieved from the FFIEC website, along

with the U.S. National Archives Catalog website (FFIEC, 2018). Additionally, historical data on Median Household Income by State was retrieved from the U.S. Census Bureau website (United States Census, 2017).

This study relies on several variables in the HMDA LAR that have been included since the 1990 FIRREA expansion.

(1) **Action Taken** indicates the disposition of a mortgage application, including *originated* (approved by the lending institution and accepted by the applicant-pair), *approved* by the lending institution but not accepted by the applicant-pair, *denied* (by the lending institution), *withdrawn* (by the applicant-pair), *closed* (by the lending institution for incompleteness), or *purchased* (by the lending institution).

(2) **Loan Purpose** indicates whether a mortgage application is intended for a *home purchase*, a *home improvement*, or a *refinance*.

(3) **Owner-Occupancy** indicates whether or not the house will be the *principal dwelling* of the applicant-pair.

(4) **Loan Type** indicates whether the mortgage is *conventional*, or *government-assisted* (FHA-insured, VA-guaranteed, or FSA/RHS-guaranteed).

(5) **Property Location** indicates the U.S. state or territory to which the mortgage application corresponds.

(6) **Applicant Race** and **Co-Applicant Race** indicate the race of an applicant as either White, Black or African American, Asian, American Indian or Alaska Native, or Native Hawaiian or Other Pacific Islander, based on the definitions provided by the U.S. Census.³

(7) **Applicant Sex** and **Co-Applicant Sex** indicate the sex of an applicant as either *Male* or *Female*.

(8) **Applicant-Pair Income** indicates the combined incomes of both applicants, including annual bonuses, measured in thousands of nominal US Dollars.

(9) **Loan Amount** indicates the quantity of credit that is pro-

³Following the 2004 expansion of the HMDA LAR, each applicant is able to identify as several races. This study only considers their primary race.

vided by the lending institution in the case of an origination, measured in thousands of nominal US Dollars. Each variable includes an option for the case in which certain parameters are not applicable, coded as “NA”.⁴

A subset of the HMDA LAR data set is constructed for the analysis using several constraints. (1) Only complete data are considered; all applications with an “NA” value for any relevant variable are removed. (2) **Action Taken** is limited to *originated* and *denied*; other cases are removed as those situations are difficult to reasonably discuss with limited data. (3) **Applicant Race** and **Co-Applicant Race** are limited to *White*, *Black*, and *Asian*, as the other two groups comprise less than one percent of the total data. (4) **Owner-Occupancy** is limited to *owner-occupied*, removing houses that are purchased to be leased to other families. The sample size for this subset is almost 96 million observations. Additionally, a subset of all originated loans is also used, which has a sample size of 78 million observations.

Several additional variables were generated using HMDA LAR data to supplement the analysis. (1) **Applicant-Pair Race** indicates the combination of Applicant Race and Co-Applicant Race. Under the assumption that the ranking of applicants is arbitrary, similar pairs are collapsed together; for example, a White-Black applicant-pair is coded identically to a Black-White applicant-pair. This variable contains six groups: *White*, *Black*, *Asian*, *White-Black*, *Black-Asian*, and *Asian-White*. (2) **Applicant-Pair Sex** indicates the combination of the of Applicant Sex and Co-Applicant Sex. Like-

⁴Several relevant variables added to the HMDA LAR in 2004 are not considered due to the time-series nature of this analysis, though they provide additional details that should be incorporated into other analyses. (1) Applicant Ethnicity and Co-Applicant Ethnicity indicates whether or not an applicant is Hispanic or Latino. (2) Property Type indicates whether a mortgage application is intended for a one-to-four family home, manufactured housing, or a multifamily home. (3) Lien Status indicates whether a mortgage is being split over several banks (secondary lien) or concentrated through a single one (first lien).

wise, Male-Female and Female-Male pairs are treated identically. This variable contains five groups: *Male-Female*, *Male-Male*, *Female-Female*, *Single*, and *Single Female*. (3) **Applicant-Pair Income Level** is a relative measure, which is approximated by comparing an applicant-pair combined income to the median household income of the Property Location in which they are applying for a mortgage. This variable contains four groups: *Low* (below 50 percent of the median income), *Moderate* (between 50 percent and 79 percent), *Middle* (between 80 percent and 119 percent), and *Upper* (120 percent or more). (4) **Origination Rate** is the percentage of loans that have been *originated* out of those that were either *originated* or *denied* by the lending institution. Other cases are not considered, as not there is not sufficient information provided in the HMDA LAR to reliably interpret these results. (5) **Loan-to-Income Ratio** is only defined for mortgages that have been originated, as Loan Amount divided by Applicant-Pair Income.

Additionally, rather than attempt to measure discrimination, which is both abstract and subjective, we will compare success in the mortgage market using a measure that we will refer to as **Advantage / Disadvantage**. We define this for any statistic as the deviation between the value for a minority group and the base group (White Male-Female conventional applicants). Specifically, any positive deviation will be denoted as *advantage*, while any negative deviation will be denoted as *disadvantage*.

III. Results

Figure 1. Average Annual Mortgage Applications by Race (1990 – 2016) [n = 95,993,618]

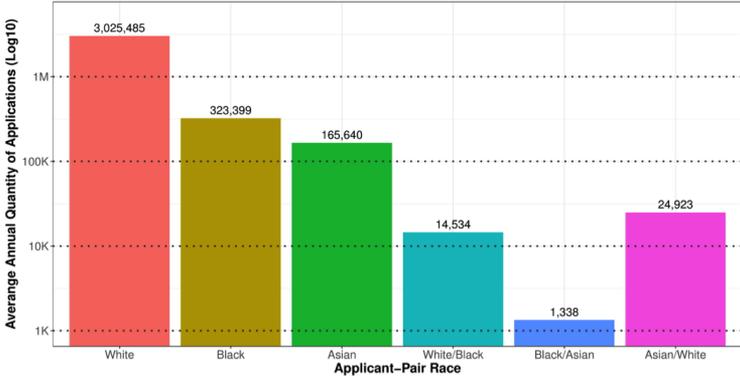


Figure 1 shows the average annual quantity of mortgage applications by race, using a log10 scale for the y-axis, to emphasize the differences in scale. Since White, Black, and Asian applicants comprise approximately 98 percent of the data, the following figures will only represent these three groups. The OLS tables, however, feature all six groups.

Figure 2. Percentage of Originated Loans with Government Assistance (1990 – 2016)

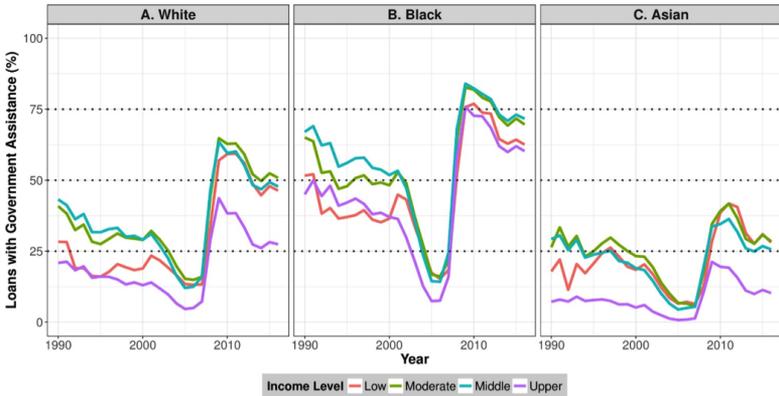


Figure 2 compares the utilization of government assistance in the mortgage process, by select Applicant-Pair Race and

Income Level groups, between 1990 and 2016. Only White, Black, and Asian applicants are included, as they comprise approximately 98% of the data, and allow for simpler graphs. When all originated loans are taken into consideration, the unadjusted average annual percentage of loans that use government assistance is 27.4% (s.d. 12.6%), for this sample; when decomposed by Race, the utilization rate is 26.4% for White applicants (s.d. 12.7%), 50.1% for Black applicants (s.d. 19.6%), and 12.5% for Asian applicants (s.d. 6.8%). Generally, Black applicants are more likely to rely on government assistance and have a higher utilization rate volatility between years, compared to other groups. When this statistic is further decomposed by Income Level, the group with the lowest utilization was Upper-Income White applicants, which have an average annual rate of 8.3% (s.d. 5.5%), while the group with the highest utilization was Middle-Income Black applicants, which have an average annual rate of 57.2% (s.d. 19.8%). When directly comparing changes in between 1990 and 2016, the utilization rate has increased by 2.9% for White applicants, 8.8% for Black applicants, and 9.8% for Asian applicants, indicating a general increase in the use of government assistance in the mortgage process.

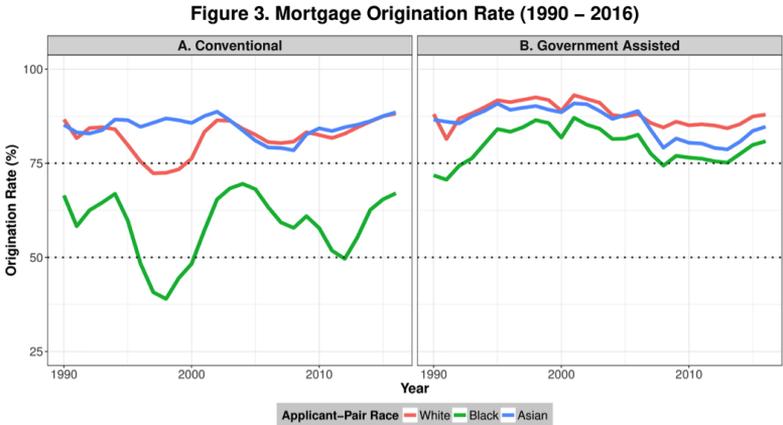


Figure 3 compares mortgage origination rates, by Loan

Type and select Applicant-Pair Race groups, between 1990 and 2016. The average annual origination rate for all conventional loans in this sample is 75.0% (s.d. 13.1%); when decomposed by Race, the average annual origination rate for conventional loans is 81.9% for White applicants (s.d. 4.5%), 58.4% of Black applicants (s.d. 8.7%), and 84.6% for Asian applicants (s.d. 2.8%). Generally, for conventional loans, Black applicants have an average annual origination rate disadvantage of 23.5%, while Asian applicants have an annual average origination rate advantage of 2.7%; further, the average annual standard deviation for is higher for Black applicants, while lower for Asian applicants, compared to White applicants. The average annual origination rate for government-assisted loans is 84.6% (s.d. 5.3%), which is 9.6% higher than the rate for conventional loans; when decomposed by Race, the average annual origination rate for government- assisted loans is 88.1% for White applicants (s.d. 3.1%), 79.7% for Black applicants (s.d. 4.6%), and 85.9% for Asian applicants (s.d. 4.1%). Generally, for government-assisted loans, both Black and Asian applicants have average annual origination rate disadvantages of 8.4% and 2.2%, respectively. When directly comparing changes directly between 1990 and 2016, the origination rate for conventional loans has increased by 1.6% for White applicants, 0.6% for Black applicants, and 3.3% for Asian applicants. Separately, the origination rate for government-assisted loans has decreased by 0.1% for White applicants, increased by 9.0% for Black applicants, and decreased by 1.9% for Asian applicants, indicating that Black applicants with government-assisted loans have benefited the most greatly during this time period.

Figure 4. Loan-to-Income Ratio (1990 – 2016)

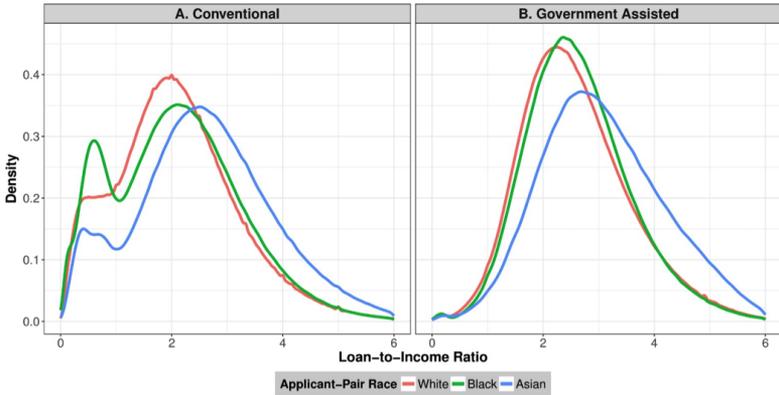


Figure 4 compares loan-to-income ratio probability densities for originated loans, by Loan Type and select Applicant-Pair Race groups. The average annual loan-to-income ratio for all conventional loans is 2.09 (s.d. 1.07); when decomposed by Race, the average annual loan-to-income ratio is 2.06 for White applicants (s.d. 1.06), 2.08 for Black applicants (s.d. 1.12), and 2.54 for Asian applicants (s.d. 1.17). Generally, for conventional loans, Black and Asian applicants have average annual loan-to-income ratio advantages of 0.02 and 0.48, respectively. Separately, the average annual loan-to-income ratio for all government-assisted loans is 2.58 (s.d. 0.94), which is 0.49 higher than that for conventional loans; when decomposed by Race, the average annual loan-to-income ratio is 2.57 for White applicants (s.d. 0.94), 2.62 for Black applicants (s.d. 0.91), and 3.02 for Asian applicants (s.d. 1.03). Generally, for government-assisted loans, Black and Asian applicants have average annual loan-to-income ratio advantages of 0.05 and 0.45, respectively. Additionally, the probability density plots for conventional loans are bimodal, with peaks roughly around 2.0 and 0.5, while the plots for government-assisted loans are unimodal, with a single peak roughly around 2.5.

Table 1. OLS Model for Origination Rate				
Variable		Estimate	Std Error	P-Value
Race				
	Black	-14.5%	0.20%	< .001
	Asian	4.8%	0.20%	< .001
	White-Black	-11.3%	0.27%	< .001
	Black-Asian	-5.7%	0.42%	< .001
	Asian-White	5.8%	0.28%	< .001
Sex				
	Male-Male	-4.2%	0.16%	< .001
	Female-Female	-3.7%	0.16%	< .001
	Single Male	1.7%	0.17%	< .001
	Single Female	4.4%	0.17%	< .001
Loan Type				
	Govt Aid	10.4%	0.19%	< .001
Loan Type * Race				
	Black	6.1%	0.28%	< .001
	Asian	-6.9%	0.29%	< .001
	White-Black	7.1%	0.39%	< .001
	Black-Asian	0.8%	0.60%	0.17
	Asian-White	-4.7%	0.41%	< .001
Income Level				
	Moderate	14.7%	0.16%	< .001
	Middle	22.6%	0.16%	< .001
	Upper	29.9%	0.16%	< .001

Table 1 presents an OLS model which predicts origination rate using Applicant-Pair Race, Applicant-Pair Sex, Loan Type, an interaction between Loan Type and Applicant-Pair Sex, Income Level, along with Location and Year (omitted).⁵⁶ The intercept is the group. White, Male-Female, Conventional, Low-Income applicants. This model predicts that for conventional loans, Black, White-Black, and Black-Asian ap-

⁵The models in Table 1 and Table 2 use Location and Year to control for differences, though the coefficients are omitted from the table as they're not relevant for the discussion. For that reason, the two models do not display the intercept, as it would be fixed to a certain Location and Year, which are arbitrary, as we're only concerned with differences in outcomes between different socioeconomic groups.

⁶Due to computational limitations, the models in Table 1 and Table 2 do not use the full data set. Rather, all applicants in each group are aggregated together for the regressions. This causes an equal-weighting problem, as states vary in population size and levels of minority disadvantage.

plicants will have disadvantages of 14.48%, 11.33%, and 5.71%, respectively, indicating that generally, even a single Black applicant in a pair decreases the likelihood of origination. Conversely, the model predicts that Asian and Asian-White applicants will have advantages of 4.82% and 5.78%, respectively, indicating that generally Asian applicants are more likely to originate a loan than White applicants. When considering Applicant-Pair Sex, the model predicts that same-sex pairs (Male- Male and Female-Female) will have disadvantages of 4.16% and 3.75%, respectively, while single applicants (Single Male and Single Female) will have advantages of 1.70% and 4.42%, respectively.

Further, when considering Loan Type, the model predicts that government assistance increases the likelihood of origination for all Race groups. Specifically, Black and White-Black applicants tend to benefit the most significantly, with predicted origination rates that are respectively 16.58% and 17.57% higher than their conventional counterparts, while Asian and Asian-White applicants tend to have the least benefit, with predicted origination rates that are only respectively 3.55% and 5.72% higher than their conventional counterparts. Additionally, the model predicts that generally, the likelihood of origination generally increases with a higher income level. This model has an adjusted R^2 value of 31.28%.

Table 2. OLS Model for Loan-to-Income Ratio				
Variable		Estimate	Std Error	P-Value
Race				
	Black	-0.02	0.01	< .001
	Asian	0.41	0.00	< .001
	White-Black	0.00	0.01	0.61
	Black-Asian	0.18	0.01	< .001
	Asian-White	0.26	0.01	< .001
Sex				
	Male-Male	-0.22	0.00	< .001
	Female-Female	-0.16	0.00	< .001
	Single Male	0.01	0.00	0.01
	Single Female	0.03	0.00	< .001
Loan Type				
	Govt Aid	0.51	0.00	< .001
Loan Type * Race				
	Black	0.05	0.01	< .001
	Asian	-0.24	0.01	< .001
	White-Black	-0.06	0.01	< .001
	Black-Asian	-0.09	0.02	< .001
	Asian-White	-0.18	0.01	< .001
Income Level				
	Moderate	-0.22	0.00	< .001
	Middle	-0.49	0.00	< .001
	Upper	-1.01	0.00	< .001

Table 2 presents an OLS model which predicts loan-to-income ratio using Applicant-Pair Race, Applicant-Pair Sex, Loan Type, an interaction between Loan Type and Applicant-Pair Race, Income Level, along with Location and Year (omitted). This model predicts that for conventional loans, Black applicants will have a disadvantage of 0.023, while Asian, Black-Asian, and Asian-White applicants will have an advantage of 0.414, 0.181, and 0.256, respectively⁶ indicating that generally, even a single Asian applicant increases the loan-to-income ratio. When considering Applicant-Pair Sex, the model predicts that same-sex pairs (Male-Male and Female-Female) will have disadvantages of 0.217 and 0.162, respectively, while single applicants (Single Male and Single Female) will have advantages of 0.011 and 0.032, respectively. Further, when considering Loan Type, the model predicts that government assistance increases the loan-to-income ratio for all Race groups. Specifically, Black, White-Black, and Asian-Black tend to benefit

the most significantly, with predicted loan-to-income ratios that are respectively 0.559, 0.445, and 0.424 higher than their conventional counterparts, while Asian and Asian-White applicants tend to have the least benefit, with predicted loan-to-income ratios that are only respectively 0.272 and 0.326 higher than their conventional counterparts. Additionally, the model predicts that generally, an applicant's loan-to-income ratio decreases with a higher income level. This model has an adjusted R^2 value of 58.41%.

IV. Discussion

This paper demonstrates that despite the numerous laws seeking to ease difficulties for minorities in the U.S. mortgage market, many groups are still highly disadvantaged relative to White Male-Female couples, the dominant group in the sample. Applicant-pairs with even a single Black applicant tend to have lower origination rates, though they benefit greatly when utilizing government assistance through FHA, VA, FSA, or RHS programs. On the other hand, Asian applicants tend to have both higher origination rates and loan-to-income ratios than White applicants, indicating that they have significant advantage, and have less need to rely on federal aid to originate their loan. Overall, Black applicants seem to be the group which benefits the most from these federal programs, though they have not been able to eliminate the White-Black origination gap. These programs do not address the underlying causes of this disparity, and therefore only provide a superficial solution.

A separate notable observation is that same-sex pairs tend to have disadvantage in regards to both their origination rates and loan-to-income ratios, and if we assume that same-sex applicant pairs are couple, it can be concluded that there may be discrimination in the housing market based on sexual orientation as well. Future studies with access to data that includes more detailed sociodemographic information may be able to test this claim more rigorously.

References

- [1] Agarwal, S., Li, S., & Mielnicki, L. (2003). Mortgages, minorities and discrimination: A Bank-specific approach. *Housing Studies*, 18(3), 295–303.
- [2] Cheng, P., Lin, Z., & Lin, Y. (2015). Racial discrepancy in mortgage interest rates. *Journal of Real Estate Finance and Economics*, 51(1).
- [3] DeSanctis, A. (2017, February 1). NAR finds real and imaginary barriers holding back prospective homebuyers. National Association of Realtors.
- [4] Faber, J. W. (2013). Racial dynamics of subprime mortgage lending at the peak. *Housing Policy Debate*, 23(2), 328–349.
- [5] Farm Service Agency. (2012). Your guide to FSA farm loans. United States Department of Agriculture. Retrieved from https://www.fsa.usda.gov/Internet/FSA_File/fsa_br_01_web_booklet.pdf.
- [6] FFIEC. (2018). HMDA & PMIC data products. FFIEC. Retrieved from <https://www.ffiec.gov/hmda/hmdaproducts.htm>.
- [7] Hanchett, T. (2000). “The other ‘subsidized housing’ federal aid to suburbanization, 1940s-1960s.” In *From Tenements to Taylor Homes: In Search of Urban Housing Policy in the Twentieth Century* (pp. 163–179). University Park, Pennsylvania: Pennsylvania State University Press.
- [8] Hilber, C. A. L., & Liu, Y. (2008). Explaining the black-white homeownership gap: The role of own wealth, parental externalities and locational preferences. *Journal of Housing Economics*, 17, 152–174.
- [9] Kau, J. B., Keenan, D. C., & Munneke, H. J. (2012). Racial discrimination and mortgage lending. *The Journal*

- of Real Estate Finance and Economics*, 45(2), 289–304.
<https://doi.org/10.1007/s11146-011-9330-3>
- [10] LaCour-Little, M. (1999). Discrimination in mortgage lending: A critical review of the literature. *Journal of Real Estate Literature*.
- [11] Lindsey-Taliefero, D. (2015). Gender differences: Mortgage credit experience. *Modern Economy*, 06(09), 977–989.
<https://doi.org/10.4236/me.2015.69093>
- [12] Munnell, A. H., Browne, L. E., McEneaney, J., & Tootell, G. M. B. (1996). Mortgage lending in Boston: Interpreting HDMA data. *American Economic Review*, 86(1), 25–53.
- [13] Robinson, J. K. (2002). Race, gender, and familial status: Discrimination in one US mortgage lending market. *Feminist Economics*, 8(2), 63–85.
- [14] Rural Development. (2017). Rural Housing Service. United States Department of Agriculture. Retrieved from <https://www.rd.usda.gov/about-rd/agencies/rural-housing-service>
- [15] Smith, M. M., & Hevener, C. C. (2014). Subprime lending over time: the role of race. *Journal of Economics and Finance*, 38(2), 321–344. <https://doi.org/10.1007/s12197-011-9220-9>
- [16] United States Census. (2017). Table H-8. Median household income by state. United States Department of Commerce. Retrieved from <https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-households.html>
- [17] United States Department of Veterans Affairs. (2018). VA home loans. United States Department of Veterans Affairs. Retrieved from <https://www.benefits.va.gov/homeloans/>

- [18] Wheeler, C. H., & Olson, L. M. (2015). Racial differences in mortgage denials over the housing cycle: Evidence from U.S. metropolitan areas. *Journal of Housing Economics*, 30(C), 33–49.

Do High Equity Valuations Predict Recessions?

Ariel Goldszmidt

Abstract

The question of whether high equity valuations indicate economic growth or foreshadow recessions has been much debated in recent years. Using autoregressive linear and logistic models, we show that high equity valuations (as measured by the cyclically-adjusted price-to-earnings ratio of the S&P 500) predict lower probabilities of recession in three months, but higher probabilities of recession in twelve months. This result seems to contradict the conclusion from a rational expectations model that high valuations predict strong dividend/earnings growth in the future. We consider the robustness of our results to model specification and time period studied, and evaluate their out-of-sample prediction performance.

I. Introduction

In the wake of the 2001 and 2008 U.S. recessions, much interest has been placed in the popular media on the role of high asset prices in financial and economic stability. The strong bull rise in equities seen over the past year has further bolstered this interest, and has raised the question of whether rapid stock market gains should be celebrated or feared. Do high equity prices signal real economic growth, or looming crashes and recessions?

Intuitive reasoning on this question is uncertain. On one hand, if market expectations are rational, equity prices equal the discounted present value of expected future dividends, meaning that high equity valuations should suggest high firm output in the future, the opposite of a recession. On the other, from a behavioral finance perspective, high valuations may be indicative of a speculative bubble caused by irrational behavioral biases, which may pop and induce a crash.

Due to this conflict, an appropriate starting point for this question is an empirical analysis. In particular, one can test whether high equity valuation in one period seem to increase, decrease, or have no effect on the probability of a recession in the next period. If high valuations are negatively correlated with future recessions, there is evidence that bull equity markets are rationally forecasting increased firm earnings and economic output. If there is no significant correlation, there is evidence that high equity valuations tell us nothing about future economic activity. Lastly, if the correlation is positive, there is evidence that high valuations may be leading indicators of future economic downturns.

One challenge to such an approach is that, even if it is the case that equity bubbles can precede or even cause crashes and recessions, it is often difficult to systematically identify bubbles in real time. How, then, shall we quantify the ambiguous term “high equity valuations”? One obvious approach is to use the price-to-earnings (PE) ratio, a classic measure of equity valuation from the value investing literature. By taking the price-to-earnings ratio of an equity aggregate, such as the S&P 500 index, one can quantify the overall valuation of the U.S. equity market. Various studies (see Section 2) have found the PE ratio to be a significant and robust predictor of future equity returns; it will be interesting, therefore, to see whether the PE ratio will have similar success in predicting recessions.

Another potential measure is the cyclically-adjusted price-to-earnings (CAPE) ratio, introduced and popularized by Robert Shiller. The CAPE ratio is calculated by dividing the current stock price by the average earnings over the past 10 years, adjusted for inflation:

$$\text{CAPE}_t = 10 \frac{\text{Price}_t}{\Pi_{t-10,t}\text{Earnings}_{t-10} + \Pi_{t-9,t}\text{Earnings}_{t-9} + \cdots + \Pi_{t-q,t}\text{Earnings}_{t-1}} \quad (1)$$

where $\Pi_{t-i,t}$ is the total gross inflation between $t-i$ and t . The CAPE ratio is designed to smooth out short-term fluctuations in earnings when assessing an equity’s valuation. The idea of smoothing a firm’s earnings over time dates back to Graham and Dodd (1934), and more recent papers (see Section 2) have

found the CAPE ratio to be an even stronger predictor of future returns than the plain PE ratio. The CAPE ratio, like the PE ratio, can be calculated for S&P 500 index to be used as a measure of valuation for the entire U.S. equity market.

The S&P 500 CAPE ratio is an especially interesting indicator because it is today at its third highest level in U.S. history, behind only its 1929 and 1999-2000 highs. Much of the current debate around the implications of high equity valuations have focused specifically on the CAPE ratio, and it therefore seems particularly relevant now to measure the predictive power of CAPE on future recessions.

In this paper, we will econometrically investigate the effect of high equity valuations on future recessions, by using linear and logistic regression models to test whether high PE or CAPE ratios increase the probability of a future recession. A more precise specification of the model is given below in Section 4.1. To test the robustness of our results, we consider an array of different model specifications and time ranges of data, and use out-of-sample testing to estimate how well our models could predict new recessions.

We begin in Section 2 with an overview of some of the important empirical and theoretical work on PE, CAPE, bubbles, and recession prediction. In Section 4.2, we use a simple asset pricing model to theoretically study the relationship between equity valuations and recessions. In Section 4, we review the data used for the analysis, and discuss in detail the development of our regression models. In Section 5, we study the results of estimating our models on the entire data set, consider their robustness, and assess the models' performance through out-of-sample testing. We conclude in Section 6 with a discussion of the potential implications of our results and possible future extensions to this research.

II. Literature Review

This paper draws primarily on three bodies of economic literature: the theoretical work on asset pricing and bubbles, the empirical finance literature on PE ratios and their predictive

powers, and the econometric literature on recession forecasting.

The theoretical work on asset price bubbles is somewhat divided. Many models operating in rational expectations frameworks—such as those of Tirole (1982) and Diba and Grossman (1985)—rule out the possibility of rational bubbles altogether. Tirole (1982) reaches this conclusion by demonstrating that rational traders will not believe they can benefit from making purely speculative trades and starting a bubble, and will not choose to enter a market in which a bubble already exists. Diba and Grossman reach the no-bubble conclusion by arguing that any rational bubble must be explosive or implosive, and then show that standard assumptions on agent behavior preclude either possibility, except in the case of fiat money.

Blanchard and Watson (1982) consider a rational expectations model in which a bubble is possible, so long as the deviation of the asset price from its “fundamental” price satisfies an appropriate stochastic growth rate. They study a few theoretical examples of such bubbles, and demonstrate how they can have real effects on other asset prices and macroeconomic variables. The authors also give empirical evidence from price innovations in the gold market that suggest violation of the no-bubble hypothesis, the rational expectations hypothesis, or both.

Several authors have also considered bubbles in behavioral models. De Grauwe and Grimaldi (2004) consider a model where agents use heuristics rather than rational expectations to evaluate the risk and return of assets. In this context, bubble equilibria can exist, and appear and disappear stochastically and unpredictably. The advantage of behavioral bubble models, the authors argue, is their ability to account for market crashes, which are difficult to explain in rational bubble models.

The empirical work on PE ratios shows more agreement in results than the theoretical bubble literature. In a study of the relationship of business cycles and market returns, Fama

and French (1989) show that the dividend yield—which is inversely proportional to the PE ratio, if we suppose dividends are proportional to earnings—is positively correlated with future market returns across a wide range of portfolios. These results confirm the intuition that when the economy is doing well and equities are expensive, returns are likely to be weak in the future, and when equities are cheap, returns are likely to be greater. Asness (2003) shows empirically that the earnings yield (the inverse of the PE) is positively correlated with and explains around 30% of variation in ten-year real equity returns. Campbell and Shiller (1998) find that dividend yields are positively correlated with future returns and PE ratios negatively correlated with future returns over a one-year time frame.

Further, Campbell and Shiller (1998) note that this correlation becomes much stronger when one replaces the standard PE with the CAPE. Several other authors have also found CAPE to be a significant forecasting variable for future financial returns. Klement (2012) finds that CAPE is a strong predictor of future returns across a wide panel of countries, including both OECD economies and emerging markets. The author also notes that the predictive power becomes still greater after accounting for differences in national interest and inflation rates. Lleo and Ziemba (2015) demonstrate that CAPE outperforms plain PE in predicting stock market crashes in the Chinese economy. These successes lead us to consider whether CAPE can also predict economic recessions.

We also draw on the existing recession prediction literature. In a seminal paper, Estrella and Mishkin (1996) study eight different potential leading indicators of recessions and offer comparisons of their lags and relative predictive powers in a probit model. They find that the slope of the yield curve at a lag of 3-4 quarters is the most effective predictor, with a pseudo- R^2 near 0.30 in sample. More recently, Hao and Ng (2011) and Hsu (2016) have used various financial markets variables, such as stock and commodity indexes, to predict recessions, and have achieved in sample pseudo- R^2 values above

0.50.

The present paper differs from earlier recession prediction work in a few important ways. We consider a wider range of macroeconomic control variables, and use monthly rather than quarterly data. Additionally, we consider linear and logistic model specifications instead of the more typical probit specification, and consider models both with and without an autoregressive term. Most importantly, we focus specifically on the role of PE and CAPE as predictors, in an attempt to learn what high equity market valuations imply about short- and long-term prospects for economic growth. As such, we use short term interest rates and long term bond yields—themselves powerful recession predictors—as controls in our models, and study how much variability in recessions PE and CAPE can explain above what is already explained by interest rates and yield curves.

III. A Simple Model of Asset Pricing with Recessions

In this section, we present a simple discrete time model of asset pricing and recessions, to provide theoretical insight into whether equity prices can be predictive of future recessions. Ours is a “rational bubbles” model based on Blanchard and Watson (1982).

A. Setup

Time is discrete, and agents are infinitely lived. Agents are risk neutral and have rational expectations. They begin with initial wealth W_0 at time 0, and in each period seek to maximize their expected wealth in the next period by investing in risky stocks and risk-free bonds. Bonds pay a rate of return $1 + r$ between t and $t + 1$. A share of stock has ex-dividend price S_t at time t , and pays a stochastic dividend d_t in each period, which we interpret as the earnings of the firm.

The economy also has a stochastic state s_t , which takes values in $\{0, 1\}$ and indicates whether the economy is in recession

in time t . We suppose further that d_t is negatively correlated with s_t , so that dividends tend to be lower in recession.

At time t , the agent solves

$$\max_{h_t, b_t} \mathbb{E}[(S_{t+1} + d_{t+1})h_t + b_t | \Omega_t] \quad \text{s.t.} \quad S_t h_t + \frac{1}{1+r} b_t = W_t$$

where h_t and b_t are holdings of stocks and bonds, respectively, $W_t = (S_t + d_t)h_{t-1} + b_{t-1}$, and Ω_t is the σ -algebra of information available at time t , which consists of the full histories of d_t and s_t up to time t . We impose no constraints on short selling.

B. Properties of Solutions

The agent's maximization problem is linear, and it must be the case that $\frac{\mathbb{E}[S_{t+1} + d_{t+1} | \Omega_t]}{S_t} = 1 + r$ in equilibrium; otherwise, the agent would choose to short an infinite quantity of the asset with lower expected return and long an infinite quantity of the asset with higher expected return, and this cannot hold in equilibrium. The equity price process S_t therefore satisfies the following recursion:

$$S_t = \frac{1}{1+r} \mathbb{E}[S_{t+1} + d_{t+1} | \Omega_t]. \quad (2)$$

By recursing this equation, applying the law of iterated expectations, and assuming the transversality condition $\lim_{t \rightarrow \infty} (1+r)^{-t} S_t = 0$, we obtain the classical solution of the stock price equaling the present-discounted value of its expected future dividends, which we call the fundamental price solution:

$$S_t^* = \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} \mathbb{E}[d_{t+i} | \Omega_t]. \quad (3)$$

As Blanchard and Watson (1982) note, however, this is not the only possible solution of equation (2). In fact, any price process of the form

$$S_t = \left[\sum_{i=1}^{\infty} \frac{1}{(1+r)^i} \mathbb{E}[d_{t+i} | \Omega_t] \right] + c_t \quad (4)$$

is a solution so long as the deviation from the fundamental price c_t satisfies $\mathbb{E}[c_{t+1}|\Omega_t] = (1+r)c_t$. The possibility of this deviation term c_t allows for the emergence of asset price bubbles in rational expectations frameworks.

The price-to-dividend ratio, which we identify with the price-to-earnings ratio of the firm, then satisfies

$$\frac{S_t}{d_t} = \left[\sum_{i=1}^{\infty} \frac{1}{(1+r)^i} \mathbb{E} \left[\frac{d_{t+i}}{d_t} \middle| \Omega_t \right] \right] + \frac{c_t}{d_t}. \quad (5)$$

Suppose the price-to-dividend ratio increases; this can happen either from an increase in the expected growth rate of the dividend, $\mathbb{E} \left[\frac{d_{t+i}}{d_t} \middle| \Omega_t \right]$, or from an increase in the relative deviation from the fundamental price, $\frac{c_t}{d_t}$. If the increase is due to an increase in the expected growth rate of the dividend, this means that dividends are expected to increase in the future; because s_t and d_t are negatively correlated, this implies an expectation of decreased probability of recession in the future. If instead the increase is due to an increase in the relative deviation $\frac{c_t}{d_t}$, we can infer no changes in the expected probability of recession, since c_t is in principle independent of s_t .

Thus, if market expectations reflect rational beliefs about the future, an increase in the price-to-dividend ratio should either be negatively correlated or uncorrelated with the probability of recession in the future, as long as the correlation between d_t and s_t is large enough.

IV. Empirical Setup

The conclusions of the previous section lead to the main empirical question of this paper: do price-to-earnings (or price-to-dividend) ratios actually correlate with probabilities of recessions in the future? Based on the model above, we expect that these may be negatively correlated.

To test this relationship, we use monthly data on the standard price-to-earnings (PE) and cyclically-adjusted price-to-earnings (CAPE) ratio of the S&P 500, as well as a monthly

indicator of US recessions. This section discusses the data and methodology of this empirical study.

A. Data

The monthly PE and CAPE of the S&P 500 going back to January 1881 have been calculated by Robert Shiller based on data from Standard and Poor's. For recession data, the St. Louis Fed's FRED website provides an indicator series of whether the U.S. was in a recession for each month since 1855, based on the NBER's US Business Cycle Expansions and Contractions data. A plot of PE, CAPE, and U.S. recessions is shown in Figure ?? in the Appendix.

In addition, it is necessary to control for other relevant macroeconomic variables in our regressions, to best isolate the relationship of PE and CAPE to recessions. To control for short- and long-term interest rates, we use the federal funds rate and the yield on constant maturity 10-year US treasuries, both of which are taken from FRED. To account for labor market conditions, we use the monthly U-3 unemployment rate, also taken from FRED. To account for consumer price conditions, we use the year-over-year U.S. inflation rate, calculated from the CPI level measured by the Bureau of Labor Statistics. To account for the potential impact of housing prices, we include the historical Real Home Price index (RHPI) calculated by Robert Shiller. Lastly, to account for leading changes in industrial output, we include the monthly percent change in the Federal Reserve's Industrial Production Index, taken from FRED.

Unfortunately, the introduction of these control variables limits the historical data we can use in our estimation. In particular, though we have PE, CAPE, RHPI, and recession data going back to the 19th Century, our data on the inflation, industrial production, unemployment, federal funds, and ten-year treasury rates go back to only 1914, 1919, 1948, 1954, and 1962, respectively. As such, we are limited in our data to considering the period 1962 to 2017, over which there were only seven recessionary periods. This limitation is especially un-

fortunate given that 21 recessions occurred between 1881 and 1962. We feel that including relevant macroeconomic variables in the analysis is worth sacrificing some data, and that predicting recent recessions is of more practical interest than predicting very early ones. However, we later consider a regression without control variables using data from this time period, to study the robustness of our results over time.

B. Model Selection

In the absence of a theoretical model providing an exact functional form to estimate, we use a multiple linear regression framework to assess the predictive power of PE and CAPE on recessions. Specifically, we consider models of the form

$$Y_t = \beta^T \mathbf{x}_t + \gamma^T \mathbf{Y}_t + \delta^T \mathbf{z}_t + \varepsilon_t \quad (6)$$

where Y_t is an indicator of the economy begin in recession at time t ; $\mathbf{Y}_t = (Y_{t-1}, \dots, Y_{t-l})^T$ is a vector of lagged values of Y_t ; $\mathbf{x}_t = (1, \text{PE}_{t-1}, \dots, \text{PE}_{t-k})^T$ is a vector of lagged values of the PE (or CAPE) ratio at time t , and a constant 1; \mathbf{z}_t is a vector of control variables; β , γ , and δ are parameter vectors; and ε_t is a random error term.

Linear probability models like the above have the advantage of being easy to evaluate and interpret, but are troublesome in that they may produce estimates outside the interval $[0, 1]$. To address this concern, we also consider logistic regression models of the form

$$Y_t = \sigma(\beta^T \mathbf{x}_t + \gamma^T \mathbf{Y}_t + \delta^T \mathbf{z}_t) + \varepsilon_t \quad (7)$$

where σ is the logistic function, $\sigma(z) = \frac{1}{1+e^{-z}}$.

One challenge in specifying time-series models like (6) and (7) is deciding how many and which lags to include. To approach this problem, we temporarily leave control variables and lagged recession indicators aside, and compare an array of linear and logistic prediction models consisting of different combinations of PE (or CAPE) lagged 3, 6, 9, and 12 months.

(Lags greater than 12 months were not significant in most regressions.) We choose the combination that gives the best fit relative to the number of variables used (with fewer variables preferred to more variables), for both the linear and logistic models. To measure fit relative to number of variables used, we use the adjusted- R^2 for the linear models, and the Akaike information criterion (AIC) for the logistic models, given by

$$\text{AIC} = 2k - \ln(\hat{L}),$$

where k is the number of parameters in the model and \hat{L} is the maximized likelihood. Lower values of AIC are preferred to higher values.

The results of our model comparison are given in Table 1 in the Appendix. In evaluating the different models, we find that models including only 3 and 12 month lags give close to the best variable-number-adjusted fit of any specification, for both PE and CAPE as well as linear and logistic models. Moreover, adding a lag of 6 or 9 months to this model had little effect on adjusted R^2 and AIC, and tended to produce coefficient estimates that were not highly significant. For these reasons, we choose to use 3 and 12 month lags of PE and CAPE in our empirical study.

Having selected lags for the main variables of interest, we turn now to determining appropriate lags for control variables and recession indicators. The inflation rate, unemployment rate, and Real Home Price Index proved to be insignificant predictors at the 5% level for all lags, and were therefore excluded from consideration. We found that both the Federal Funds rate and the ten-year treasury yield were most significant at lags of 6 months in all regressions, and that including multiple lags of these variables did not improve model performance, so we choose to include only 6 month lags of these variables in the regression. Similarly, industrial production proved to be significant only at a 3 month lag, and is included in the final regressions as such. Lagged values of the recession indicator proved to be significant at 3, 6, and 9 month lags across all models; we choose to include only a 6 month lag, which is able

to capture the persistence effect of recessions without dominating the other variables in the regressions. However, we will also consider models with no lagged values of the recession indicator, to assess the performance of PE and CAPE as future predictors in the absence of this autoregressive component.

V. Results

A. Full Data Set Estimation

The preceding discussion leads to the regression equation

$$Y_t = \beta_0 + \beta_1 PE_{t-3} + \beta_2 PE_{t-12} + \gamma Y_{t-6} + \delta_1 FFR_{t-6} + \delta_2 TYT_{t-6} + \delta_3 PRO_{t-3} \quad (8)$$

where FFR is the federal funds rate, TYT is the ten-year treasury yield, and PRO is the percent change in the industrial production index. We also consider the equation with PE replaced by CAPE, and the equivalent logistic regression equations for both variables.

In addition to these four regression equations, we also consider the equivalent equations with the autoregressive recession indicator term Y_{t-6} removed, for a total of eight equations to estimate.

We estimated all eight specifications on the data set described in Section 4.1, which consists of monthly readings on all variables from January 1962 to October 2017. The results of the estimations for the linear and logistic models are given in Tables 2 and 3 in the Appendix. Since the usual measures of goodness of fit— R^2 , adjusted R^2 , F -test—are only defined for OLS, we include for the logistic models the Akaike information criterion (AIC) and the McFadden’s pseudo- R^2 , calculated as

$$R_{\text{McFadden}}^2 = 1 - \frac{\ln(L_M)}{\ln(L_0)}, \quad (9)$$

where L_M is the likelihood of the fitted logistic regression on all independent variables and L_0 is the likelihood of a fitted logistic regression on only a constant. A more detailed account of this statistic can be found in Domencich and McFadden (1975).

In our results, we find that the R^2 and adjusted R^2 values of the linear models are all relatively high, and suggest that lagged PE/CAPE, short- and long-term interest rates, and changes in the industrial production index explain between 33 and 44% of variation in the recession indicator. A regression of the recession indicator on only the lagged values of the controls (FFR, TYT, and PRO) without any PE or CAPE terms produced a high R^2 of 30%, consistent with earlier findings in the literature that the width of the yield curve (TYT – FFR) is a powerful predictor of recessions. The inclusion of PE or CAPE terms in the model therefore seems to explain an additional 3-14% of the variation in the recession indicator, on top of what can be explained by the control variables alone.

We also notice a distinct ordering of the adjusted R^2 values of the linear models and McFadden’s pseudo- R^2 values for the logistic models: those of the autoregressive models are all greater than those of the equivalent non-autoregressive model, which suggests that the autoregressive Y_{t-6} term is indeed a significant predictor of future recessions. This result is in line with the intuitive notion that recessions are persistent, at least on short time scales: moving from a recession month to another recession month is more likely than moving from a non-recession month to a recession month.

More interestingly, the R^2 and adjusted R^2 of the linear CAPE regressions are 6-10 percentage points higher than those of the equivalent PE regressions, and the pseudo- R^2 of the logistic CAPE regressions are 14-19 percentage points higher than those of the equivalent PE regressions, for both the autoregressive and non-autoregressive models. These results suggest that CAPE is a substantially more powerful predictor of future recessions than the regular PE, just as it is also a more powerful predictor of long-run market returns.

The individual parameter estimates also support this conclusion. All eight regression specifications suggest that the coefficients on $CAPE_{t-3}$ and $CAPE_{t-12}$ differ significantly from 0 at the 0.1% level. The same is not true of the PE terms: in both the linear and logistic autoregressive models (column

(3) in the tables), the coefficient on PE_{t-3} does not differ significantly from 0, while the coefficient on PE_{t-12} differs significantly from 0 only in the logistic specification. In the non-autoregressive PE models (column (4)), the coefficient on PE_{t-3} differs significantly from 0 while the coefficient on PE_{t-12} does not, in both the linear and logistic models.

The estimated coefficients on the control variables and the autoregressive term in all regression specifications were all significant at the 5% level, and most were significant at the 0.1% level. These coefficients were all more significant in the linear models than in the logistic models. The signs on all these coefficients are consistent across all eight models, and their magnitudes are mostly consistent between the four linear models and separately between the four logistic models; this suggests that our coefficient estimates are at least somewhat robust to model specification. The signs of these coefficients are all in line with intuition: the coefficient on Y_{t-6} is positive, capturing the persistence effect of recessions; the coefficient on PRO_{t-3} is negative and large in magnitude, suggesting that increasing industrial production dramatically decreases the probability of recession; and the coefficients on FFR_{t-6} and TYT_{t-6} are positive and negative, respectively, and of similar magnitude, suggesting that a flattening yield curve is a leading indicator of a recession.

The estimated coefficients on the CAPE terms also show consistent signs between the linear and logistic models, and comparable magnitudes between the autoregressive and non-autoregressive model. In the autoregressive models, the coefficients estimated on $CAPE_{t-3}$ and $CAPE_{t-12}$ are -0.038 and 0.039 respectively in the linear model, and -0.636 and 0.624 in the logistic model. The interpretation of these numbers is that a one point increase of CAPE decreases the probability of recession in 3 months by 3.8% and increases the probability of recession in 12 months by 3.9%, or decreases the log odds of recession in 3 months by 0.636 and increases the log odds of recession in 12 months by 0.624. Interestingly, the results show that a high CAPE ratio predicts a lower probability of

recession in the short term, but a higher probability of a recession in the longer term. The results make some intuitive sense: economies do not collapse immediately in the midst of an equity boom, but a large rise in valuations may be followed later by a large contraction, which can be coincident with the start of a recession.

Figures 2 and 3 in the Appendix show the probabilities of recessions in each month implied by the fitted linear autoregressive and logistic autoregressive models, respectively, overlaid with bars indicating actual recession periods. Overall, both fits appear quite strong: the model appropriately gives high recession probabilities in 1970, 1973, 1981, 2001, and 2008. Both models fare worst on explaining the 1991 recession. For the most part, both models also give low recession probabilities to non-recessionary periods; the greatest exception is 1967, in which both models assign a nearly 50% probability of recession. Of course, however, the models' performance can only meaningfully be assessed through out-of-sample testing; we perform these tests in Section 5.2 below.

As an additional test of the robustness of our estimated coefficients on the lagged values of CAPE, we also ran linear and logistic regressions of Y_t on $CAPE_{t-3}$, $CAPE_{t-12}$, and Y_{t-6} using data from 1881 to 1961. This data had to be excluded from the main regressions due to the nonavailability of data on the control variables. The results of these regressions are shown in Table 4 in the Appendix. We find that both models estimate coefficients of the same sign and comparable magnitude to those of the models estimated on data from January 1962 to October 2017. This result suggests that our findings on the relationship between CAPE and future recessions are robust over a wide range of time periods.

B. Out-of-Sample Testing

To properly measure the model's overall predictive power, it is necessary to use out-of-sample testing. This proceeds as follows: suppose a recession occurred between times t and $t+m$. We remove the data between t and $t+m$ from the data set,

fit the model on the remaining data, and see what probability the fitted model would assign to a recession at time t based on the data from $t - 12$ to $t - 3$. Repeating this procedure on every recession, as well as a few randomly selected non-recession periods, provides a quantitative assessment of how accurately the model can predict recessions on which it has not been fit.

The results of this out-of-sample testing for six recession months and five non-recession months are given in Table 5. We see that out-of-sample performance is far from perfect, but still reasonable: predicted recession probabilities for actual recession months average in the 30-35% range, while probabilities for non-recession months average in the 1-2% range. The average squared errors of the linear and logistic models are very close, but that of the linear model is slightly smaller, suggesting that it may have a slight edge in out-of-sample performance.

Overall, both models seem biased toward predicting against recessions, by giving probabilities that tend to skew toward 0 rather than 1. This is to be expected, as there are many more non-recessionary periods than recessionary periods in our data set, which leads our model fittings to prefer type II over type I errors. This bias could be corrected by giving higher weight to months in recession periods, rather than weighting each month equally.

We also notice that both models performed quite poorly in predicting the 1990 and 2007 recessions. These recessions were most connected with changes in oil and housing prices, respectively, rather than equity prices, and as such it is not surprising that CAPE fails to predict them. In future research, it may be fruitful to study more general recession prediction models based on the valuations of an array of investments, including housing and commodities in addition to equities.

Overall, the out-of-sample testing suggests that both the linear and logistic models can function at least somewhat well in forecasting unseen recessions, suggesting that the CAPE ratio indeed contains economically meaningful information about

future recession probabilities. Moreover, we found that the out-of-sample regressions all produced coefficient estimates and R^2 values close to those of the full data set regressions, further supporting the robustness of the results described in Section 5.1.

VI. Conclusion

While a simple rational expectations model suggests that high equity valuations should correlate with lower risk of recession, empirical evidence points to a more complex relationship. We estimate that a one point increase in the S&P 500 cyclically-adjusted price-to-earnings (CAPE) ratio tends to predict an approximately 4% lower probability of recession in three months, but an approximately 4% higher probability of recession in twelve months. The regular PE ratio, however, is not a significant predictor. These results seem robust to model specification and time period studied. A high and peaking CAPE ratio, then, appears to be a leading indicator of economic downturns.

There are many possible extensions to this work. First, it will be necessary to develop a theoretical model that can account for the observed CAPE-recession relationship; it may be necessary to introduce behavioral asset pricing models to do so. Additionally, there are many other macroeconomic indicators—such as consumer confidence, business owner surveys, and commerce indexes—we may include in the regressions, to sharpen our estimates of the direct effects of CAPE. It would also be valuable to include measures of valuation of other assets, such as commodities and housing, to account for the diversity of market crashes and recessions observed throughout history. Lastly, it would be interesting to see whether a similar CAPE-recession relationships hold in different countries, using the CAPE values of regional stock indexes in place of the S&P 500.

References

- [1] Asness, C. S. Fight the fed model. *The Journal of Portfolio Management* 30, 1 (2003), 1124.
- [2] Blanchard, O., and Watson, M. Bubbles, Rational Expectations and Financial Markets. NBER Working Paper.
- [3] Campbell, J., and Shiller, R. Valuation Ratios and the Long-Run Stock Market Outlook. *The Journal of Portfolio Management* 24, 2 (1998), 1126.
- [4] Campbell, J. Y., and Shiller, R. J. Stock Prices, Earnings, and Expected Dividends. *The Journal of Finance* 43, 3 (1988), 661676.
- [5] De Grauwe, P., and Grimaldi, M. Bubbles and Crashes in a Behavioural Finance Model. CESifo Working Paper Series No. 1194.
- [6] Diba, B., and Grossman, H. The Impossibility of Rational Bubbles. NBER Working Paper.
- [7] Domencich, T. A., and McFadden, D. *Urban Travel Demand - A Behavioral Analysis*. North-Holland Publishing Company, 1975.
- [8] Estrella, A., and Mishkin, F. S. Predicting U.S. Recessions: Financial Variables as Leading Indicators.
- [9] Fama, E. F., and French, K. R. Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics* 25, 1 (1989), 2349.
- [10] Filardo, A. J. The 2001 US Recession: What Did Recession Prediction Models Tell Us? *SSRN Electronic Journal* (2004).
- [11] Graham, B., and Dodd, D. *Security Analysis*. McGraw-Hill, 1934.

- [12] Hao, L., and Ng, E. C. Predicting Canadian recessions using dynamic probit modelling approaches. *Canadian Journal of Economics/Revue canadienne d'conomique* 44, 4 (2011), 1297-1330.
- [13] Hsu, T. U.S. recession forecasting using probit models with asset index predictor variables.
- [14] Klement, J. Does the Shiller-PE Work in Emerging Markets? *Credit Suisse* (2012).
- [15] Lleo, S., and Ziemba, W. T. Predicting Chinese Stock Market Crashes. *SSRN Electronic Journal* (2015).
- [16] Shiller, R. Price Earnings Ratios as Forecasters of Returns: The Stock Market Outlook in 1996.
- [17] Tirole, J. On the Possibility of Speculation under Rational Expectations. *Econometrica* 50, 5 (1982), 1163–1181.

Appendix

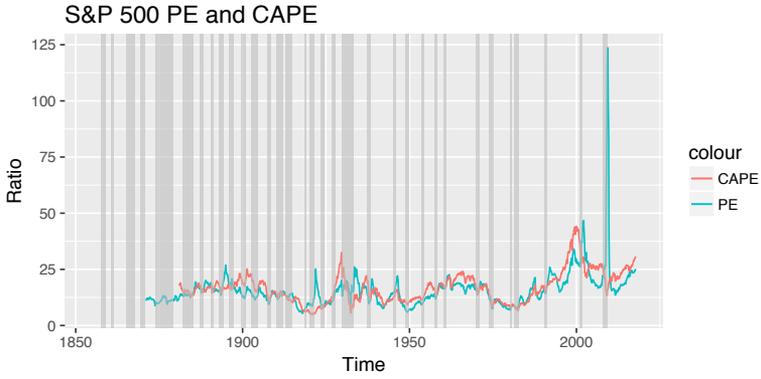


Figure 1: Monthly values of the S&P 500 price-to-earnings (PE) ratio and ten-year cyclically-adjusted price to earnings (CAPE) ratio. Both series calculated by Robert Shiller. Grey bars indicate recessions, as determined by NBER's US Business Cycle Expansions and Contractions data.

$t-3$	$t-6$	$t-9$	$t-12$	PE Adjusted R^2	PE AIC	CAPE Adjusted R^2	CAPE AIC
✓				0.015	503.7	0.037	473.4
	✓			0.021	495.2	0.016	489.8
		✓		0.018	491.0	0.004	500.0
			✓	0.013	491.2	0.000	503.1
✓	✓			0.020	470.0	0.116	395.1
✓		✓		0.020	483.9	0.179	325.4
✓			✓	0.019	488.7	0.193	313.3
	✓	✓		0.020	492.2	0.094	407.4
	✓		✓	0.021	493.2	0.124	392.2
		✓	✓	0.017	492.3	0.048	466.5
✓	✓	✓		0.019	471.7	0.178	327.0
✓	✓		✓	0.020	472.0	0.194	313.5
✓		✓	✓	0.019	485.9	0.199	308.0
	✓	✓	✓	0.020	494.0	0.126	389.1
✓	✓	✓	✓	0.020	472.9	0.198	309.9

Table 1: Comparison of fit of different model specifications. Checkmarks indicate which lagged values of PE or CAPE were included in linear and logistic regressions of a recession indicator. Adjusted R^2 values are from the linear regressions, while AIC (Akaike Information Criterion) values are from the logistic regressions.

	<i>Dependent variable: Y_t</i>			
	(1)	(2)	(3)	(4)
CAPE $_{t-3}$	-0.038*** (0.004)	-0.044*** (0.004)		
CAPE $_{t-12}$	0.039*** (0.004)	0.044*** (0.004)		
PE $_{t-3}$			0.0005 (0.001)	0.003** (0.001)
PE $_{t-12}$			0.002* (0.001)	0.002 (0.001)
FFR $_{t-6}$	0.063*** (0.006)	0.064*** (0.006)	0.076*** (0.007)	0.083*** (0.007)
TYT $_{t-6}$	-0.052*** (0.009)	-0.051*** (0.009)	-0.065*** (0.008)	-0.063*** (0.009)
Y $_{t-6}$	0.188*** (0.033)		0.270*** (0.036)	
PRO $_{t-3}$	-5.229*** (1.458)	-7.395*** (1.440)	-7.901*** (1.532)	-11.211*** (1.527)
Constant	0.097* (0.058)	0.132* (0.059)	0.071 (0.044)	0.017 (0.045)
R^2	0.449	0.423	0.372	0.319
Adjusted R^2	0.444	0.418	0.366	0.313

Note:

*p<0.05; **p<0.01; ***p<0.001

Table 2: Results of estimating linear models of Y_t (a recession indicator variable) on lagged values of CAPE (S&P 500 cyclically-adjusted price-to-earnings ratio), PE (S&P 500 price-to-earnings ratio), FFR (federal funds rate), TYT (ten year treasury yield), Y , PRO (percent change in the industrial production index), and a constant. Time measured in months. Data consists of monthly readings of all variables for the U.S. from January 1962 to October 2017. Standard errors in parentheses.

	<i>Dependent variable: Y_t</i>			
	(1)	(2)	(3)	(4)
CAPE $_{t-3}$	-0.636*** (0.098)	-0.700*** (0.095)		
CAPE $_{t-12}$	0.624*** (0.089)	0.677*** (0.086)		
PE $_{t-3}$			0.018 (0.011)	0.036*** (0.011)
PE $_{t-12}$			0.028 (0.018)	0.022 (0.020)
FFR $_{t-6}$	0.644*** (0.110)	0.610*** (0.104)	0.892*** (0.121)	0.819*** (0.107)
TYT $_{t-6}$	-0.407** (0.155)	-0.343* (0.144)	-0.815*** (0.150)	-0.634*** (0.126)
PRO $_{t-3}$	-31.342 (23.304)	-49.114* (22.180)	-69.916** (21.549)	-106.221*** (21.094)
Y_{t-6}	0.926* (0.444)		2.009*** (0.406)	
Constant	-4.443*** (1.128)	-4.350*** (1.121)	-3.540*** (0.755)	-4.036*** (0.767)
Akaike Inf. Crit.	228.965	231.297	299.956	323.186
McFadden's Pseudo- R^2	0.570	0.561	0.428	0.377

Note:

*p<0.05; **p<0.01; ***p<0.001

Table 3: Results of estimating logistic regression models of Y_t (a recession indicator variable) on lagged values of CAPE (S&P 500 cyclically-adjusted price-to-earnings ratio), PE (S&P 500 price-to-earnings ratio), FFR (federal funds rate), TYT (ten year treasury yield), Y , PRO (percent change in the industrial production index), and a constant. Time measured in months. Data consists of monthly readings of all variables for the U.S. from January 1962 to October 2017. Standard errors in parentheses.

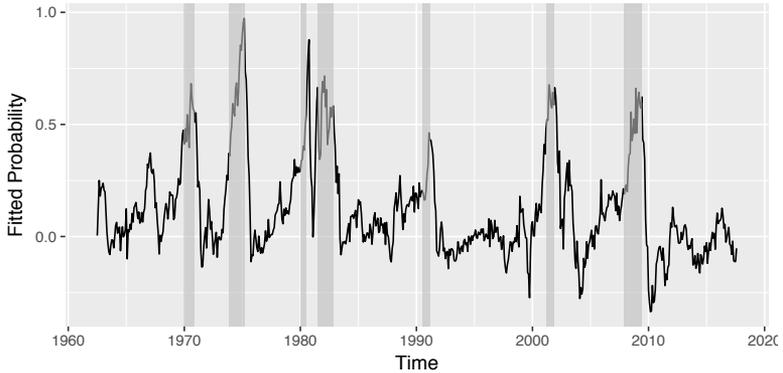


Figure 2: Fitted probability of recession in month t based on CAPE at $t - 3$, CAPE at $t - 12$, the Federal Funds rate and the ten year treasury yield at $t - 6$, the percent change in the industrial production index at $t - 3$, and a recession indicator at $t - 6$. Fitted using a linear regression model. Actual NBER recessions indicated by gray bars.

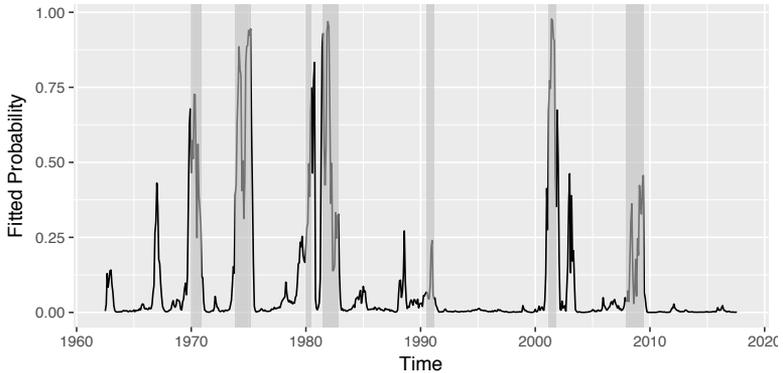


Figure 3: Fitted probability of recession in month t based on CAPE at $t - 3$, CAPE at $t - 12$, the Federal Funds rate and the ten year treasury yield at $t - 6$, the percent change in the industrial production index at $t - 3$, and a recession indicator at $t - 6$. Fitted using a logistic regression model. Actual NBER recessions indicated by gray bars.

	<i>Dependent variable: Y_t</i>	
	Linear Regression	Logistic Regression
	(1)	(2)
$CAPE_{t-3}$	-0.054*** (0.006)	-0.355*** (0.041)
$CAPE_{t-6}$	0.068*** (0.006)	0.432*** (0.042)
Y_{t-6}	0.334*** (0.029)	1.686*** (0.164)
Constant	0.053 (0.048)	-2.401*** (0.318)
R^2	0.297	
Adjusted R^2	0.295	
Akaike Inf. Crit.		950.484
McFadden's Pseudo- R^2		0.450

Note: *p<0.05; **p<0.01; ***p<0.001

Table 4: Results of estimating linear and logistic regression models of Y_t (a recession indicator variable) on lagged values of CAPE (S&P 500 cyclically-adjusted price-to-earnings ratio), Y , and a constant. Time measured in months. Data consists of monthly readings of all variables for the U.S. from January 1881 to December 1961. Standard errors in parentheses.

Month	Predicted Probability of Recession	
	Linear Model	Logistic Model
November 1973	0.329	0.332
January 1980	0.257	0.162
July 1981	0.568	0.870
July 1990	0.189	0.057
March 2001	0.388	0.697
December 2007	0.173	0.036
July 1965	0.120	0.012
July 1976	-0.046	0.005
January 1986	0.059	0.016
January 1995	0.008	0.012
January 2015	-0.024	0.003
Average Squared Error	0.266	0.280

Table 5: Out-of-sample predicted probabilities of recession in month t , based on CAPE at $t - 3$, CAPE at $t - 12$, the Federal Funds rate and the ten year treasury yield at $t - 6$, the percent change in the industrial production index at $t - 3$, and a recession indicator at $t - 6$. Fitted using linear and logistic models, on data sets in which each recessionary period was removed. For the non-recessionary periods (the last five rows), two years after the given month was removed from the fitting data. Average squared error calculated by averaging the squared difference of the predicted probability and the actual recession indicator.

Does Snowfall Influence Ski Visitation to Resorts in Colorado? An Empirical Analysis Uncovering the Relationship

Alex Vergara

Advisor: Dr. Jonathan Hughes

Abstract

The skiing industry is perceived as heavily influenced by weather conditions and snowfall totals. This analysis serves as an empirical work highlighting the lack of correlation between ski visits and snowfall to 15 resorts in Colorado from 1995-2004. Lowess curve graphs, ordinary least squares, fixed effects, quantile snowfall tests, and snowfall thresholds were the econometric methods used that yielded no convincing evidence. Rather, a strong relationship is drawn between ski visits and other variables such as number of trails, snowmaking capabilities, acreage of resort and income by county. This paper strives to apply meaning to such results.

I. Introduction

The Colorado ski industry is the largest of any state in the United States, responsible for \$4.8 billion in economic value and supportive of 46,000 jobs in 2015 (Blevins, 2016). With more than 25 ski resorts to explore and increased affordability of multi-mountain season passes such as the Epic Pass and Rocky Mountain Super Pass, Colorado leads the United States in annual ski visits year after year. The industry is strong and lucrative but is there reason to believe this success will deteriorate? Climate change is a looming question mark and poses extreme threats to the industry. It is well documented that climate change alters snow patterns in dramatic manners, leaving some ski seasons mildly dry and others severely wet (Yang, Wan 2010). Given out of state skiers represent 55-60% of total visits to resorts in Colorado, it is plausible that with decreased snowfall totals, such a demographic will choose to ski

elsewhere. My motivation in writing this thesis was to analyze the correlation between ski visits and snowfall and hypothesize whether the potential decreases in snowfall may have any effect on future visits to resorts. If so, climate change could have catastrophic consequences to one of the most important economic sectors of Colorado.

As an athlete on the club ski team at University of Colorado, I spend much of my free time traveling and competing at different ski resorts across the United States during the winter months. Such experiences have allowed me to observe the demographics and preferences that attract specific skiers to specific resorts. It is a common conception in the ski industry that wherever there is more snow, there will be more people skiing but during my travels I did not find this to be necessarily true. Rather, I observed there to be other variables to be more important in determining what resort a skier may go to. Quantifying the other variables became my second motivation in writing this analysis.

II. Literature Review

In order to construct a rich and differentiated analysis, a robust literature review was conducted. Published econometric papers surveying the correlation between snowfall and ski visits were scarce but nonetheless evident. Two particular papers, “The Demand for Winter Sports: Empirical Evidence for the Largest French Ski Lift Operator” and “Climate Change and Aspen: An Assessment of Impacts and Potential Responses” were found to be the most relatable as each seek to understand the correlation between ski visitation and snowfall. “Climate Change and the Ski Industry in Eastern North America: A Reassessment” was a published economics paper which quantified the importance of snowmaking machines and is also relevant to the thesis.

“The Demand for Winter Sports: Empirical Evidence for the Largest French Ski Lift Operator” is a published econometric paper by economist Martin Falk of the Austrian Institute of Economic Research (Falk, 2015). Falk attempts to uncover

the determinants of long run winter tourism exclusively at ski resorts in France. Data is compiled from aggregated ski visits to six resorts apart of the Compagne des Alpes from the years 1993-2011. He mentions previous research has failed to incorporate a measure of income and lift ticket prices compared to competitors, a critical component in determining ski demand (Falk, 2015). With a nuanced approach, Falk's methodology finds merit in using real GDP figures from foreign countries as a substitute for income as it provides an accurate measure of purchasing power. The economist's rationale is where someone skis is greatly attributable to their income and given French ski resorts are frequented by tourists, real GDP accounts for a tourist's purchasing power. Utilizing ordinary least squares and fixed effect regressions in log-linear format: snowfall, temperature, real GDP and relative lift ticket prices serve as the independent variables. Although Falk finds that snowfall is statistically significant, the degree to which it influences skier visitation is small. OLS results show a snowfall coefficient of 0.025 meaning a 1% change in snowfall results in a 0.025% change in ski visits (Falk, 2015). When accounting for fixed effects, a 1% change in snowfall results in a 0.016% change in ski visits (Falk, 2015). Another conclusion Falk makes is that skiing at the sampled resorts is a luxury good, exhibiting characteristics of increased demand as income rises. Falk's research raises value in testing income and other variables associated with luxury goods in the ski industry (fine dining, ski-in ski-out lodging).

"Climate Change and Aspen: An Assessment of Impacts and Potential Responses" is an economic analysis sponsored by the Aspen Global Change Institute which strictly looks at climate change prediction models and the effects to the ski industry in Aspen (Gosnell, Travis, Williams, 2006). Forecasted measures of ski visitation are the main focus but restaurant sales, housing value, summer tourism and job displacement are all analyzed as well. To recapture the report by AGCI, a correlation between skiing and snowfall is drawn using data from 1966-2005 across the four ski resorts in Aspen. Using

the Intergovernmental Panel on Climate Change low, medium and high impact emission assumptions, predicted number of skier days and socioeconomic effects varied annually for years up to 2100 (Gosnell et al., 2006). Key findings included: decreases in snowfall yielded small decreases in skier visitation, a heightened reliance on snowmaking machines over the years surveyed, and an expected increase in rain rather than snow up to year 2100 (Gosnell et al., 2006). Given Gosnell’s focal point of climate change mitigation, variables such as number of trails, skiable acreage, snowmaking capabilities, and peak elevation were all examined. Such variables will also be tested in this paper.

“Climate Change and the Ski Industry in Eastern North America: A Reassessment” published by economists Daniel Scott and Brain Mills analyzed the effectiveness of snowmaking machines at ski resorts in Vermont, Ontario, Quebec and Michigan (Scott, Mills 2006). The research takes six resorts that recently added snowmaking capabilities and makes predictions to the benefits of the machines. It is forecasted that between 2020-2030 under the highest climate change thresholds generated by the Canadian Climate Impact Scenario, five of the six resorts have reductions of skiable days of less than 25% (Scott, Mills 2006). The economists calculated that resorts without snowmaking capabilities would lose 32-65% of skiable days. (Scott, Mills 2006). This stark contrast stresses the importance in testing snowmaking significance in my analysis.

III. Data

The data sets and variables chosen in this paper try to understand the correlation between visits and snowfall. Data compiled for this paper takes form in both panel data and cross-sectional data. Both forms have variables which seem plausible in identifying a skier’s preferences. Ski visits and snowfall to the fifteen resorts, county income and county population exhibit panel data as they vary both with time and by resort. The other fifteen variables that stay constant with time

and only vary by resort are considered cross-sectional data. These variables include: skiable acreage, vertical drop, peak elevation, distance from Denver, distance from closest airport, number of lifts, snowmaking acres, number of trails, mountain lodging (dummy), off-mountain dining/shopping (dummy), nightlife (dummy), night skiing (dummy), season pass usage for different mountains (dummy), and upscale amenities (dummy). Cross-sectional data comes from 2016 levels as data from 1995-2004 could not be found. These variables also theoretically change with time but corresponding data to match changes across all fifteen resorts could not be found. The variables listed with dummy in parenthesis are interaction terms which a ski resort either has or does not have the variable. In an econometric sense, a one would be given to a resort that has the dummy and a zero would be given to the resort that doesn't have the dummy.

Data for majority of the listed variables was published by companies that cater information to the ski community; Powder Hounds, On the Snow and Colorado Ski Country are some of the publications used. The data that pertains to demographics was published by different governmental agencies of Colorado. Specifically, per county income was published by the Colorado Department of Labor and per county population data was obtained from the Colorado Department of Local Affairs. Each ski resort will receive income and population levels based on the county the ski resort is located in.

Ski visitation data was the largest obstacle to starting the analysis as majority of the ski resorts keep such information confidential. Numerous unsuccessful attempts were made either through in-person meetings or phone calls to acquire ski visit data directly from the resort. A useful data set was recommended by a University of Colorado graduate who was interested in predicting pollution levels at ski resorts in Colorado (Shelesky, 2016). Published by Colorado Ski Country, ski visits to each of the fifteen Colorado ski resorts sampled from 1995-2004 were recorded (Mills, 2004). These resorts include: Arapahoe Basin, Aspen Mountain, Beaver Creek, Brecken-

ridge, Copper Mountain, Crested Butte, Keystone, Loveland, Monarch, Steamboat, Snowmass, Telluride, Vail, Winter Park and Wolf Creek. The snowfall data was acquired from a publication called Best Snow and has season totals of snowfall for all resorts from 1995-2004 (Crocker, 2017). This paper finds Best Snow's snowfall data to be more pertinent to the analysis than snowfall data from weather collection websites such as the National Organization of Atmospheric Administration based in Boulder, Colorado. For purposes of less measurements across Colorado ski resorts and snowpack as the only statistical measure, NOAA was found to be an inferior indicator for skiing purposes.

IV. Methodology

Lowess curve graphs, ordinary least squares, fixed effects, quantile regression, and snowfall thresholds were the econometric methods found to be most applicable to the analysis. This section highlights the econometric equations used and their interpretations.

A. Lowess Curve Graphs

A lowess curve graph allows a linear regression line to be drawn that tries to best fit the data points. By taking into account the sum of squares, the lowess line equally weighs all data points and plots a constant slope. The lowess curve graph is most effective in nonparametric scenarios because the distribution shape is unknown. In the application of snowfall analysis, this paper finds strong evidence of unknown parameters therefore requiring a nonparametric estimator. The four graphs used include: ski visits and snowfall, natural log of ski visits and natural log of snowfall, ski visits and snowfall for the year 1999, and ski visits and snowfall exclusive to Crested Butte Mountain Resort. The first two show an overall relationship between ski visits and snowfall where the latter two provide the reader with a more in-depth analysis of the non-linear relationship.

B. Ordinary Least Squares

In ordinary least squares regression, the unknown parameters are estimated through the minimization of sum of squares; by doing so, OLS finds a linear relationship between the variables. For purposes of this paper, multiple OLS regressions were utilized. Below are the equations:

$$\begin{aligned} \ln(V_{it}) = & \beta_0 + \beta_1 \ln(Snfwl_{it}) + \beta_2 \ln(Pop_{ct}) \\ & + \beta_3 \ln(Inc_{ct}) + \sum_{j=1}^{15} \beta_j \ln(X_i) + \epsilon_{it} \quad (1) \end{aligned}$$

$$\begin{aligned} \ln(V_{it}) = & \beta_0 + \beta_1 \ln(Snfwl_{it}) + \beta_2 \ln(Acre_i) \\ & + \beta_3 \ln(Snwmkg_i) + \beta_4 \ln(Trls_i) + \epsilon_{it} \quad (2) \end{aligned}$$

Equation 1 is simply used as a glimpse into how correlated all variables are in relation to ski visits. The equation serves as a stepping stone and highlights where further analysis needs to be dedicated. In the equation, the subscripts i , t , and c highlight variables that vary with time and by county/resort. Σ represents the fifteen control variables listed in the data section of this paper and are believed to also influence skier preferences in choosing ski resorts. Lastly, ϵ is the error term and accounts for observational error in the regression. Each variable expressed in the equation is in natural log format as resorts vary by visits drastically. Monarch and Vail, for instance, highlight the disparity in size of how many people annually ski at each resort; the former attracts around 300,000 people annually over the sampled timeframe while the latter attracts 1,500,000. To claim a one inch increase in snow would impact ski visits to each resort by the same amount would be false. It is for this reason that natural logs are used as the method accounts for percentage change in skier visits relative to the visitation size of each resort. Equation 2 also uses natural logs and incorporates control variables. Such variables include resort acreage, snowmaking acreage and number of trails and are used to prevent omitted variable bias. For a credible econometric test, variables which strongly correlate to visits need

to be included in the regression. Resort acreage, snowmaking acreage and number of trails all exhibit a strong correlation to visits, thus their inclusion. The main objective of equation 2 is to understand how a one percent change in snowfall results in x percent change in ski visits. This is known as the ski visit elasticity with respect to snowfall.

C. Fixed Effects

Given some of the data varies by resort and time, a more rich and robust econometric test requires fixed effects to be used. This model accounts for differences amongst resorts by holding constant all data that varies by resort or with time. By doing this, all fifteen ski resorts can be equally compared as differences in snowfall, population and income amongst resorts are held constant. Below are the equations:

$$\ln(V_{it}) = \beta_0 + \beta_1 \ln(Snfwl_{it}) + \beta_2 \ln(Pop_{it}) + \beta_3 \ln(Inc_{it}) + \delta_i + \epsilon_{it} \quad (3)$$

$$\ln(V_{it}) = \beta_0 + \beta_1 \ln(Snfwl_{it}) + \beta_2 \ln(Inc_{ct}) + \delta_i + \epsilon_{it} \quad (4)$$

$$\ln(V_{it}) = \beta_0 + \beta_1 \ln(Snfwl_{it}) + \beta_2 \ln(Inc_{ct}) + \delta_i + \delta_t + \epsilon_{it} \quad (5)$$

Equation 3 highlights all variables that exhibit panel data characteristics (snowfall, population and income) and serves as a general sampling of the statistical relationships. Resort fixed effects are accounted for by δ_i and the error term is accounted for by ϵ_{it} . For reasons mentioned above, natural logs will be used to account for size differences across resorts. Equation 4 offers more detailed insight to the relationship between ski visits and snowfall by only relying on income as a control variable. Income shows a correlation to ski visits in fixed effect analysis so we use the variable to eliminate omitted variable bias. Finally, equation 5 is beneficial to the analysis by accounting for resort fixed effects δ_i and time fixed effects δ_t . Considering each of these effects, every individual year can be tested for statistical significance. This paper understands that there may not be a holistic relationship between visits and snowfall so using a time fixed effect yields more detailed findings.

D. Quantile Snowfall Test

A quantile snowfall test also seeks to find a deeper understanding of the relationship between ski visits and snowfall. Because the timeframe between 1995-2004 is ten years and there are fifteen resorts sampled, 150 snowfall data points exist. In essence, the 150 snowfall data points are assigned into six different groups based on mean snowfall. These groups are categorized as: severely dry, moderately dry, mildly dry, mildly snowy, moderately snowy and severely snowy. Each of the six groups is then tested to measure if snowfall influences ski visits. In this circumstance, a quantile snowfall test is valuable because it replaces the holistic test of ski visits and snowfall with more specific subcategory tests. In the equation, snowfall varies with time by resort, hence the subscripts i and t . Snowfall is used as the independent variable and q highlights the six different quantiles, one being severely dry and six being severely snowy. ϵ is the error term and accounts for observational error in the regression; δ_i is the resort fixed effect. Natural log of income is used to omit variable bias.

$$\ln(V_{it}) = \beta_0 + \beta_1 \ln(Snfwl_{it}) + \beta_2 \ln(Inc_{ct}) + q1_{it} + q2_{it} + q3_{it} + q4_{it} + q5_{it} + q6_{it} + \delta_i + \epsilon_{it} \quad (6)$$

E. Snowfall Threshold

A snowfall threshold to test statistical significance serves as a lower bound effect. In application, a snowfall is chosen and all data points less than the minimum threshold are used in the regression. For instance, if a threshold of 220 inches were chosen, 55 of 150 snowfall recordings would be used in the regression as this is how many data points fall below the 220-inch threshold. Snowfall thresholds for below average snowfall ($x < 200$ in), average snowfall ($240 < x < 280$ in) and above average snowfall ($x > 350$ in) are tested individually for significance. A quantile snowfall test highlights a range of snowfall whereas a snowfall threshold tests an exact inch level of snowfall. The equation takes into account resort fixed effects δ_i and is in natural logs to offer a fair comparison across resorts.

Income is once again used to omit variable bias.

$$\ln(V_{it}) = \beta_0 + \beta_1 \ln(Snfwl_{it}) + \beta_2 \ln(Inc_{ct}) + \delta_i + \epsilon_{it} \quad (7)$$

V. Results

Strong statistical evidence found no relationship between ski visits and snowfall. To start, flat lowess curve lines would elicit a lack of relationship. P-values in fixed effect regressions were too high to identify any type of correlation and correctly reject the null hypothesis that snowfall does not influence ski visits. More refined statistical models such as quantile tests of snowfall and snowfall thresholds yielded results that substantiate the conclusions made in fixed effects. Below is a thorough investigation of the results organized by econometric method.

A. Lowess Curve Graphs

In Figure 8, the relationship between ski visits and snowfall highlights the flat nature of the lowess slope. In the graph, it is apparent that some of the most visited resorts are during times when there is the least amount of snow. The same can be said when there is above average snow and below average ski visits. These outliers further elude to the lack of a relationship. Figure 9 reveals more insightful data as the variables visits and snowfall are regressed with natural logs, allowing for a fairer comparison across resorts. This graph shows the ski visit elasticity with respect to snowfall and similar to Figure 8, a flat lowess line shows no relationship. If there was any statistical relationship we would expect an increase in natural log of visits to be matched by an increase in natural log of snowfall. Such a relationship would be shown through a positive and increasing lowess line. This, however, is lacking. Figure 10 and figure 11 are graphs which plot ski visits and snowfall for the year 1999 and ski visits and snowfall to Crested Butte across 1995-2004, respectively. Neither show any relationship as data points appear to be nonlinear and show no correlation.

B. Ordinary Least Squares

Figure 12 displays the statistical results of all nineteen variables in equation 1's regression. Snowfall in particular is of interest as a p-value of 0.100 and coefficient of -0.069 is recorded. In this scenario, the p-value implies we can reject the null hypothesis that snowfall has no effect on ski visits with 90% confidence. The natural log coefficient implies a 1 percent change in snowfall results in a -0.069 percent decrease in visits. Despite the reporting of statistical significance at a credible level, I find less merit in the test because of the high number of control variables. With more variables used in the regression, less observations per variable are used, manipulating the results. Ultimately, the motive for the test is to gather a preliminary notion of what variables need further analysis. Snowmaking acreage, terrain acreage and number of trails all show a strong correlation to visits and thus are used in equation 2. These variables are relied upon to prevent omitted variable bias and give a more accurate interpretation of snowfall and ski visits. In figure 13, snowfall again portrays statistical significance with a p-value of 0.076 and coefficient of -0.141 . Despite statistical significance, the OLS model does not account for fixed effects, an important critique to the results.

C. Fixed Effects

Holding differences across resorts constant, fixed effects yielded no correlation between ski visits and snowfall. When looking at figure 14, neither natural log of snowfall nor natural log of population are even slightly significant in relation to visits but natural log of income is. The natural logs of snowfall, population and income each exhibit p-values of 0.448, 0.331, 0.077 and coefficients of -0.03 , 0.12 and -0.12 , respectively. Because income displays a correlation to ski visitation, the variable must be utilized for omitted variable bias purposes moving forward. In figure 15, the natural log of snowfall and natural log of income show no statistical significance. The p-value of snowfall decreases relative to the previous test but not to a

level which highlights a noteworthy relationship with ski visits (0.212). The coefficient at this level is -0.04 meaning a 1 percent increase in snowfall results in a -0.04% decrease in ski visits. Given the p-value, however, the coefficient is irrelevant as the confidence interval is between a negative and positive number. The final fixed effect result, expressed in figure 16, draws the relationship between ski visits and snowfall by year with income as a control variable. In practice, this could give a richer testament to which year, if any, portrays some type of relationship but the resulting p-value of snowfall shows a lack of significance to visits when looking at the relationship annually. The coefficients by year highlight how many more people on average skied during that year. We know this because the time fixed effect is accounted for in the regression.

D. Quantile Snowfall Test

Consistent with fixed effect results, no statistical correlation is found between ski visits and snowfall when looking at the six quantiles grouped by mean snowfall. The most interesting finding in figure 17 is the result for quantile 6. By definition, this quantile reflects a heavy snowfall season; one that is considerably above average. A p-value of 0.800 shows relative to the rest of the quantiles, heavy snowfall leads to the least responsive behavior of skiers. The notion that wherever there is more snow, more people will be skiing is proven false by this quantile.

E. Snowfall Threshold

Snowfall thresholds for below average snowfall ($x < 200$ in), average snowfall ($240 < x < 280$ in) and above average snowfall ($x > 350$ in) found no significance in relation to snowfall as well. This test served as a lower bound effect, discovering if a minimum threshold of snow influenced skiers. Inch increases in snowfall were tested individually in below average, average and above average groups. Findings included a lack of relationship between ski visits and snowfall for almost all levels

tested. There would be instances in which x amount of snowfall would have a relationship with ski visits but this was rare. I hypothesize that because an incremental increase in snowfall may coordinate with 2-5 more data points used in the regression, overweighting the relationship became a byproduct. Figure 18, 19 and 20 shows three graphs which highlight below average (180 inch), average (270 inch) and above average (380 inch) thresholds, respectively.

VI. Conclusion

Based on the analysis provided, no statistical relationship was apparent between ski visits and snowfall. Lowess curve graphs, ordinary least squares, fixed effects, quantile snowfall tests, and snowfall thresholds were all used as econometric methods to find a correlation but yielded no convincing evidence. This paper shows there should be considerable doubt that the Colorado ski industry will be impacted by changing snowfall patterns in future years. The lack of correlation between ski visits and snowfall implies future snowfall volatility should be met unresponsively by Colorado skiers. Previous literature finds a relationship between the variables but one that is extremely small. Reasons why Falk and Gosnell find a relationship between ski visits and snowfall exploit the limitations in this paper's analysis. For one, no data was used that conveyed snow conditions per day and a skier's responsiveness. This would offer a more accurate demonstration of ski visits and snowfall and could drastically differentiate results. Given how hard it was in this research to obtain aggregated ski visits, data on daily ski visits and daily snow conditions seems unfeasible. Second, the timeline used in this paper is only ten years which is relatively short. For a more robust analysis, more data points which include ski visits and snowfall over a longer period of sampled years may benefit the results. Because the sampled timeframe is only from 1995-2004, the analysis may be drawing from an anomaly in the ski industry in Colorado. Finally, data used in this analysis does not account for season pass holders. Over the past ten years, season passes to multiple

mountains may have drastically changed skier behavior. Because multi-mountain season passes are affordable and owned by many skiers, preferences in ski mountains and their respective snow conditions may have changed. Moving forward, this paper finds merit in examining other variables that influence ski visits such as skiable acreage, snowmaking acreage, number of trails and income by county. Each of the variables showed a strong correlation to ski visits and would be interesting to examine further in a different analysis.

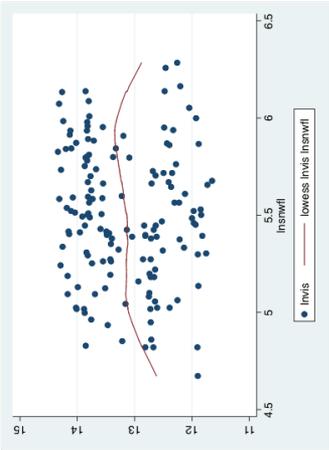
References

- [1] Blevins, Jason. "Skiers' Economic Impact in Colorado Adds up to \$4.8 Billion." *The Denver Post*, Vail Resorts, 19 Apr. 2016.
- [2] Burakowski, Elizabeth, and Matthew Magnusson. "Climate Impacts on the Winter Tourism Economy in the United States." *National Resource Defense Council* (2012): 1-32. Web. 2 Apr. 2017.
- [3] Butsic, Van, Ellen Hanak, and Robert G. Valletta. "Climate Change and Housing Prices: Hedonic Estimates for Ski Resorts in Western North America." *Federal Reserve Bank of San Francisco* (2009): 1-44. Web. 2 Apr. 2017.
- [4] Mellanie, Mills. "Annual Colorado Skier Visits 1993-2004." *Colorado Ski Country*. Sept. 2004. Raw data. Denver.
- [5] "Colorado." *On The Snow*. Mountain News Corporation, 2017. Web. 06 May 2017.
- [6] Crocker, Tony. "Ski Season Summary." *Best Snow*. N.p., 1 Jan. 2017. Web. 04 May 2017.
- [7] "Facts & Stats." Colorado Ski Country. *Colorado Ski Country*, 2017. Web. 04 May 2017.

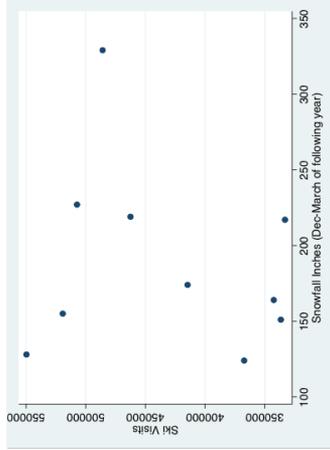
- [8] Falk, Martin. "The Demand for Winter Sports: Empirical Evidence for the Largest French Ski- Lift Operator." *Tourism Economics* 21.3 (2015): 561-80. Web. 2 Apr. 2017.
- [9] Gosnell, Hannah, Bill Travis and Mark Williams. "Climate Change and Aspen: An Assessment of Impacts and Potential Responses." *Aspen Global Change Institute* (2006): 1-139. Web. 2 Apr. 2017.
- [10] Scott, Daniel, and Brian Mills. "Climate Change and the Ski Industry in Eastern North America: A Reassessment." (2006): 90-96. Web. 2 Apr. 2017.
- [11] Shelesky, Stephen. *Examining the Economic Impacts of Climate Change on Colorado Ski Communities Through 2050*. Thesis. University of Colorado, 2016. Boulder: CU Scholar, 2016. Print.
- [12] "Ski Resorts USA Data." *Powder Hounds*. N.p., 2015. Web. 1 Apr. 2017.
- [13] USA. Department of Labor and Employment. Colorado. *Personal Income in Colorado by County*. N.p.: n.p., 2017. *Colorado Information Marketplace*. Web. 20 Apr. 2017.
- [14] USA. Department of Local Affairs. Colorado. *Demographic Profiles by County*. N.p.: n.p., 2017. Colorado Portal. Web. 25 Apr. 2017.
- [15] "What's New at Vail Over the Past 5 Years." *Vail*. Vail Resorts Management Company, 25 Oct. 2013. Web. 06 May 2017.
- [16] Yang, Jianming, and Chunyan Wan. "Progress in Research on the Impacts of Global Climate Change on Winter Ski Tourism." *Advances in Climate Change Research*, vol. 1, no. 2, Nov. 2010, pp. 55-62.

Appendix

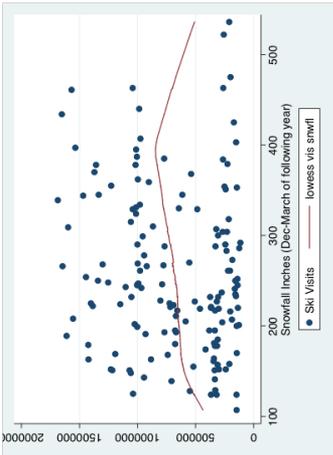
Lowess Curve Graphs



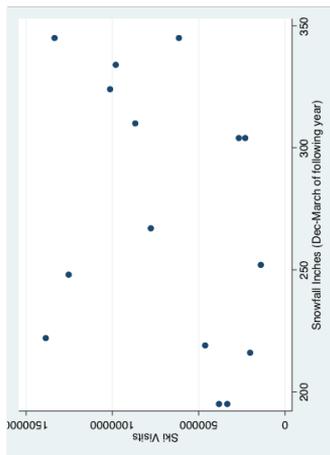
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11



8



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OLS Results

lnvis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	-.0690488	.041629	-1.66	0.100	-.1514443	.0133467
lnpop	-.1376124	.0616132	-2.23	0.027	-.2595621	-.0156626
lninc	-.0366383	.0661224	-0.55	0.581	-.1675131	.0942365
lnacre	.6674236	.0597203	11.18	0.000	.5492205	.7856267
lnvert	-.1832511	.1856077	-0.99	0.325	-.5506207	.1841186
lnpk	-3.826718	.4745679	-8.06	0.000	-4.766021	-2.887415
lndisden	.0100247	.1001929	0.10	0.920	-.1882851	.2083345
lndisair	-.3694147	.0881183	-4.19	0.000	-.5438255	-.1950038
lnlfts	.5554207	.1245901	4.46	0.000	.3088221	.8020193
lnsnwmkg	.2917173	.0832827	3.50	0.001	.1268776	.456557
lntrls	-.7083222	.1723429	-4.11	0.000	-1.049437	-.3672074
1.out	-.34089	.2155209	-1.58	0.116	-.7674663	.0856863
1.shp	0 (omitted)					
1.ngtlf	0 (omitted)					
1.dtw	.4758291	.0534595	8.90	0.000	.3700178	.5816403
1.sznp	1.001163	.1365337	7.33	0.000	.7309241	1.271401
1.upscl	-.7185331	.096542	-7.44	0.000	-.9096166	-.5274495
_cons	48.9761	5.140959	9.53	0.000	38.80071	59.1515

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lnsnwfl	-.1410691	.0788798	-1.79	0.076	-.297069	.0149309
lnacre	.660711	.0681427	9.70	0.000	.5259457	.7954762
lnsnwmkg	.2213685	.0298743	7.41	0.000	.1622862	.2804507
lntrls	.6251308	.150254	4.16	0.000	.3279746	.9222871
_cons	4.751417	.5627272	8.44	0.000	3.638516	5.864319

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Fixed Effect Results

lnvis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	-.0302399	.0397692	-0.76	0.448	-.1089072	.0484275
lnpop	.1229316	.1208636	1.02	0.311	-.1161486	.3620117
lninc	-.1289062	.0724329	-1.78	0.077	-.2721856	.0143732
_cons	13.54006	1.15494	11.72	0.000	11.25547	15.82465

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lnvis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	-.045935	.0366581	-1.25	0.212	-.1184433	.0265733
lninc	-.0927086	.0630952	-1.47	0.144	-.2175085	.0320913
_cons	14.41644	.769167	18.74	0.000	12.89506	15.93782

15

lnvis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	-.0084967	.0453674	-0.19	0.852	-.0982915	.081298
lninc	-.0131362	.1190401	-0.11	0.912	-.2487499	.2224776
Year						
1996	-.0058747	.0423123	-0.14	0.890	-.0896226	.0778733
1997	.0405392	.0458144	0.88	0.378	-.0501403	.1312187
1998	.0597533	.0487283	1.23	0.222	-.0366936	.1562001
1999	.0356774	.0478452	0.75	0.457	-.0590216	.1303763
2000	-.0585737	.0543623	-1.08	0.283	-.1661719	.0490245
2001	.0209328	.060438	0.35	0.730	-.098691	.1405566
2002	-.0449207	.0524234	-0.86	0.393	-.1486813	.0588398
2003	.0406692	.0560099	0.73	0.469	-.0701901	.1515286
2004	.0118252	.0553971	0.21	0.831	-.0978211	.1214715
_cons	13.34885	1.276593	10.46	0.000	10.82211	15.87558

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Quantile Snowfall Results

lnvis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	.0678053	.5186501	0.13	0.896	-.9575305	1.093141
lninc	1.099212	.2475215	4.44	0.000	.6098794	1.588546
q1	-.2179397	.5239104	-0.42	0.678	-1.253675	.8177954
q2	-.2751353	.4448127	-0.62	0.537	-1.1545	.604229
q3	-.1179705	.3756914	-0.31	0.754	-.8606866	.6247457
q4	-.2215738	.3260494	-0.68	0.498	-.8661511	.4230036
q5	-.1329226	.2992488	-0.44	0.658	-.724517	.4586717
q6	.067215	.2649307	0.25	0.800	-.4565348	.5909648
_cons	1.120833	3.910698	0.29	0.775	-6.610349	8.852015

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Snowfall Threshold Results

Invis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	-.0582237	.0477806	-1.22	0.225	-.1527385	.0362912
lninc	-.0905554	.0635202	-1.43	0.156	-.2162046	.0350938
_ID180_1	-.0157514	.0391029	-0.40	0.688	-.0931008	.061598
_IResort_2	.2968588	.0551151	5.39	0.000	.1878357	.4058818
_IResort_3	.9827861	.0531735	18.48	0.000	.8776037	1.087969
_IResort_4	1.74597	.0521442	33.48	0.000	1.642824	1.849116
_IResort_5	1.345068	.0522042	25.77	0.000	1.241803	1.448333
_IResort_6	.5202179	.0631811	8.23	0.000	.3952395	.6451963
_IResort_7	1.527054	.054391	28.08	0.000	1.419464	1.634645
_IResort_8	.0110982	.0527777	0.21	0.834	-.0933013	.1154977
_IResort_9	-.5633945	.0615067	-9.16	0.000	-.6850608	-.4417282
_IResort_10	1.125371	.0533588	21.09	0.000	1.019822	1.23092
_IResort_11	1.444341	.0525707	27.47	0.000	1.34035	1.548331
_IResort_12	.3050855	.0554988	5.50	0.000	.1953034	.4148677
_IResort_13	1.885004	.0526068	35.83	0.000	1.780943	1.989066
_IResort_14	1.400577	.0540613	25.91	0.000	1.293638	1.507515
_IResort_15	-.4187729	.0584577	-7.16	0.000	-.534408	-.3031378
_cons	13.69038	.7894301	17.34	0.000	12.12881	15.25195

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Invis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	.0146783	.052842	0.28	0.782	-.0898483	.1192049
lninc	-.0998228	.0629005	-1.59	0.115	-.2242461	.0246005
_ID270_1	.055125	.0347923	1.58	0.115	-.0136977	.1239477
_IResort_2	.3009101	.0541944	5.55	0.000	.1937082	.408112
_IResort_3	.9855405	.0519236	18.98	0.000	.8828304	1.088251
_IResort_4	1.73665	.0520136	33.39	0.000	1.633762	1.839538
_IResort_5	1.34182	.0517259	25.94	0.000	1.239501	1.444139
_IResort_6	.5147514	.0621912	8.28	0.000	.3917311	.6377717
_IResort_7	1.524894	.0533416	28.59	0.000	1.419379	1.630409
_IResort_8	.0135349	.0522584	0.26	0.796	-.0898372	.1169071
_IResort_9	-.5716711	.0612048	-9.34	0.000	-.6927403	-.4506018
_IResort_10	1.119947	.0529756	21.14	0.000	1.015156	1.224738
_IResort_11	1.440803	.0518743	27.77	0.000	1.33819	1.543416
_IResort_12	.2970886	.0545909	5.44	0.000	.1891024	.4050747
_IResort_13	1.885189	.0521333	36.16	0.000	1.782064	1.988313
_IResort_14	1.398396	.0535281	26.12	0.000	1.292512	1.50428
_IResort_15	-.4252364	.058089	-7.32	0.000	-.5401422	-.3103307
_cons	13.35401	.7947077	16.80	0.000	11.78199	14.92602

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lnvis	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnsnwfl	-.0576463	.0440225	-1.31	0.193	-.1447272	.0294347
lninc	-.0998657	.0649852	-1.54	0.127	-.2284128	.0286814
_ID380_1	-.0194164	.0401473	-0.48	0.629	-.0988317	.0599989
_IResort_2	.2925997	.0544826	5.37	0.000	.1848276	.4003717
_IResort_3	.9892078	.0526695	18.78	0.000	.8850223	1.093393
_IResort_4	1.748348	.0523793	33.38	0.000	1.644736	1.851959
_IResort_5	1.345129	.0521525	25.79	0.000	1.241966	1.448292
_IResort_6	.5127554	.0633626	8.09	0.000	.3874179	.6380929
_IResort_7	1.52134	.0540462	28.15	0.000	1.414431	1.628249
_IResort_8	.0099229	.0526517	0.19	0.851	-.0942275	.1140732
_IResort_9	-.56626	.0618498	-9.16	0.000	-.688605	-.443915
_IResort_10	1.127331	.0532887	21.16	0.000	1.021921	1.232741
_IResort_11	1.449433	.0524406	27.64	0.000	1.3457	1.553166
_IResort_12	.3082203	.0545533	5.65	0.000	.2003084	.4161322
_IResort_13	1.88545	.0525798	35.86	0.000	1.781442	1.989458
_IResort_14	1.397807	.0540975	25.84	0.000	1.290797	1.504817
_IResort_15	-.4210789	.0587745	-7.16	0.000	-.5373407	-.3048171
_cons	13.80143	.8466194	16.30	0.000	12.12674	15.47613

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The Transferability of Human Capital: A Case Study of Refugees in the United States

Sarang D Murthy[†]

Abstract

The earnings and employment outcomes of working age refugees in the U.S. were analyzed and compared with the earnings and employment outcomes of working age native-born Americans. The analysis attempts to find the extent to which human capital is transferable between national borders, specifically for refugees entering the U.S. The findings suggest that refugees are likely to earn sizably less and be closer to the poverty level when compared to their native counterparts i.e., two identical individuals, one a refugee and the other a native, see considerable earnings differentials when controlling for observable measures of human capital.

I. Introduction

The conventional definition of human capital refers to the stock of personal abilities, experiences, intelligence, training, and judgment that enhances the productivity of a worker in order to increase economic output. Traditionally, education, training and health have been considered the most important investments in human capital (Becker, 1964) – they are viewed as essential inputs in bettering a worker’s capabilities. Nobel Memorial Prize recipient, Gary S. Becker popularized Human

[†]Special thanks to Dr. Jane E. Knodell, for her invaluable guidance, critique, and support, and to Dr. Emily Beam for her assistance with the regression models. Thanks also goes to Dr. Nathalie Mathieu-Bolh and Dr. Stephanie Seguino for their comments and suggestions, as well as Dr. Pablo Bose for his insights on refugee assimilation.

Acknowledging, also: Denise Lamoureux, Vermont State Refugee Coordinator, Dr. Shawna Shapiro, Middlebury College, Natalie Meyer, ROC Inc., and Kevin Stapleton, Economist, Vermont Department of Labor. This research was supported by the University of Vermont’s College of Arts & Sciences’ APLE Summer Stipend.

Capital Theory and was a pioneer in the study of human capital.

Becker does acknowledge that, apart from the three pillars of human capital i.e., education, training, and health, “No discussion of human capital can omit the influence of families on the knowledge, skills, health, values, and habits of their children” (Becker, 1993), but finds that the positive relation between the earnings of parents and children is not as strong as the relation between the years of schooling of parents and their children. Becker finds that the continuing growth in the per capita income of many countries during the nineteenth and twentieth centuries is partly due to the expansion of scientific and technical knowledge that raises the productivity of labor and other inputs in production (Becker, 1975). The earnings of more-educated people are almost always well above average, though the gains are generally larger in less-developed countries.

Although Becker’s insights into human capital will long be remembered in the economics literature as pioneering, the shortcomings of the theory must be discussed in depth in order to paint a holistic picture of human capital. The broad applications and even broader implications of the flaws in the theory in relation to its geographical inequalities and distribution, gender and gender identity wage differentials, racial and sexual discrimination in the work place, socio-economic and socio-cultural status, and earnings differentials as a result of immigration, sought and forced, are flaws in traditional Human Capital Theory that must be addressed. For the scope of this paper, we will focus our attention on the last-mentioned shortcoming; this paper will consider the case of Refugees in the U.S, in order to explain the *Transferability of Human Capital Amongst Refugees*.

To do this, we assess the breadth of Human Capital Theory that defines human capital as the stock of personal abilities, experiences, intelligence, training, and judgment that enhances productivity of a worker in order to increase economic output. In doing so, we seek to analyze the earnings and employment

outcomes of working age refugees in the United States. Our cross-sectional analysis will look at data from the three-year American Community Survey (ACS). The ACS contains 36 months of collected data, and this paper uses the 2005-2007 and 2009-2011 surveys. It is to be noted that this is not panel data, but we focus on these two periods in time in order to capture the unemployment and earning disproportionalities, if any, between refugee workers and native-born workers, before the Great Recession and during it.

Resettlement to the United States is a long process that can take months, or even years. The U.S. resettlement process involves a series of reviews conducted by a number of agencies. The Obama-era White House stated that “refugees undergo more rigorous screening than anyone else allowed into the United States.”¹

With the integration resources available to refugees, it may not be unfair to assume that they should be able to successfully use the human capital they have accumulated in their respective countries of origin in order to find and keep employment in the host country i.e., the U.S. Unfortunately, existing data pertaining to refugees and their economic outcomes is scant. We borrow ideas in the existing literature to extract refugee data from data on immigrants as a whole; our methodology for data extraction will be explained in length in later sections.

Having a low level of education in America may mean that you have a higher chance of being unemployed (see appendix, table I). But, oftentimes refugees do not have low levels of education yet see higher levels of unemployment.² Seventy-five percent of refugee adults in the 2009-2011 period had at least a high school education, higher than the 68 percent rate for other immigrants but lower than the 89 percent rate for U.S. born adults. Twenty-eight percent of refugee adults had at least a four-year college degree, roughly equivalent to both the 29 percent attainment of U.S.-born adults and 27 percent at-

¹Pope, 2015; See Appendix for the screening process for refugee entry into the U.S.

²Messick, 2012, and Vijaya et al, 2017

tainment of other immigrants with four-year college degrees.³

The link between earnings and educational attainment is positive; the median yearly income for people 25 and older by educational attainment in the period 2002 and 2014 (taken as an average) is as follows (see appendix, table IIa & IIb): \$25,644 for an individual with a less than high school education, \$35,820 for a high school graduate, \$41,476 for an individual with some college, and finally \$63,712 for an individual with a bachelor's degree or higher. If Human Capital Theory holds true, these numbers should be similar for our population in question: refugees. In the following sections, this paper is going to attempt to understand the earnings outcomes for refugees using cross-sectional regression models for two periods in times, namely, prior to the 2008 recession and during it. We will compare the outcomes for refugees against the outcomes for native-born Americans. The layout of the coming sections is as follows:

II) A brief literature review of previous critiques of traditional Human Capital Theory, III) a specific literature review of research that looks at the portability of human capital with regard to immigrants entering Israel, IV) the presentation of the data and an explanation of the sources, V) the presentation of the regression model and our assumptions, VI) the presentation of our findings from the model, VII) our conclusion, a discussion of the limitations, and ideas for future research in the field, VIII) bibliography, and finally IX) the appendix containing all regression tables and other charts.

II. Literature Review: Extensions and Explanations of Human Capital Theory

In discounting the complexities of the human experience, Human Capital Theory (HCT) fails to explain a host of possible factors that influence a worker's earning potential. As discussed above these may include: geographical equity and distribution, gender and gender identity wage differentials, racial

³Capps et al, 2015

and sexual discrimination in the work place, socioeconomic status, and immigration, both sought and forced. Following Becker's theory of human capital, the literature in this field has attempted to distinguish between "specific" and "general" human capital. General human capital (such as literacy, race, ethnicity, etc.) is useful to all employers, whereas, specific human capital refers to skills or knowledge useful only to a single employer or industry. Economists have tried to build on the classical HCT for years to make the theory more robust. Below are some relevant academic pieces of literature that have hoped to achieve this goal.

One of the most widely known advances in traditional HCT is the Signaling Theory.⁴ The Signaling Theory provides another explanation for obtaining higher wages. It suggests that "the education levels of individuals indicate certain innate characteristics such as their propensity to be intelligent, their dedication, time management skills, and ability to follow instructions."⁵ This suggests that education does not lead to increased human capital, but rather acts as a mechanism by which workers with superior innate abilities can signal those abilities to prospective employers and so gain above average wages. Therefore, even if education doesn't necessarily improve the productivity of a worker, signaling is a way to avoid market inefficiencies associated with the problem of adverse selection.

Duleep and Regets (1999) look at immigrants and their patterns of human capital investment in the country they have newly emigrated to. They hypothesize that "immigrants will invest more in human capital than natives if their cost of investment is lower or their return greater."⁶ They consider the aggregate stock of human capital as a function of the initial stock before immigration and the stock of human capital produced or procured in the host country. Having controlled for initial levels of human capital, their model predicts "a high

⁴Spence, M., 1973

⁵Ibid.

⁶Ibid.

rate of investment and earnings growth for immigrants with low initial skill transferability.”⁷ The authors also find that immigrants are more likely than the native-born population to undertake investment in their specific human capital as the perceived opportunity cost is much lower in the country of settlement.

A Marxian critique of HCT was undertaken by Bowles & Gintis (1975). In their paper, the authors comment that neo-classical economics has long considered labor as a good and the exchange of labor for a wage as a goods market. They lament that, “human capital theory is the most recent, and perhaps ultimate, step in the elimination of class as a central economic concept.”⁸ Their critique is not based on the commonly recognized shortcomings of HCT namely, market imperfections such as monopolies and labor unions that drive a wedge between wages and marginal product. Instead, the critique is that “the theory of human capital offers no theory of reproduction at all and presents a very partial theory of production, one which abstracts from the social relations of production in favor of technical relations.”⁹ They assert that macroeconomic considerations, market structure, technical change, economic dualism, and other presumably central aspects of the distribution problem are ignored. In Marxian fashion, they conclude their critique with the following: “[The Human Capital Theory] framework provides an elegant apology for almost any pattern of oppression or inequality (under capitalism, state socialism, or whatever), for it ultimately attributes social or personal ills either to the shortcomings of individuals or the unavoidable technical requisites of production. It provides, in short, a good ideology for the defense of the status quo.”¹⁰

In what seems to be an applicable modeling of Bowles & Gintis’ (1975) Marxian critique, Summerfield et al. (2015) hypothesize that “industry wage differentials may result either

⁷Spence, M., 1973

⁸Bowles and Gintis, 1975

⁹Ibid.

¹⁰Ibid.

from the structure of the industry (demand type) or human capital (supply type) characteristics of the employed labor force.”¹¹ Their study uses data from the U.S. and Germany to analyze the effect of demand and supply characteristics on average earnings across industries. What they found, most importantly, was that worker ability was not the whole story behind industry earnings differentials. Institutional characteristics and macroeconomic factors played an important, if not substantial, role, as well. That is to say, workers and their abilities (human capital) were not the only leading indicators to the industry wage differentials.

Another notable market imperfection, and the crux of our analysis, is the portability (referred to as transferability in this paper), or lack thereof, of human capital between man-made borders. Even though intellect cannot be quantified between two people, let alone between two nations, “the national origin of an individual’s human capital is a crucial determinant of its value.”¹² In keeping with this presented market imperfection, the following is a literature review of Rachel Friedberg’s findings amongst Israeli emigrants in terms of the portability of their human capital. Friedberg’s research is most closely associated with the theme of our own research: the *transferability of human capital amongst refugees*.

III. Literature Review: The Portability of Human Capital

Friedberg (2000) finds that variation in the return to foreign schooling across origin countries may reflect differences in its quality and compatibility with the host labor market (in her model, Israel is the host market). She explains that there might be multiple country-specific skills that the natives have an obvious advantage in but holding these factors constant, an immigrant with the exact educational skill set is likely to earn less than his/her domestic counterpart. Foreign human capi-

¹¹Summerfield et al., 2015

¹²Friedberg, 2000

tal almost always “earns a lower return than domestic human capital.”¹³

Most immigrants complete some schooling in their countries of origin. Friedberg finds, however, that many “immigrate at young ages and obtain virtually all their human capital after moving to the host country.”¹⁴ Thus, a sizable number of immigrants have accumulated some combination of foreign and domestic human capital. Previous work on immigrant and native earnings has allowed the return to human capital to differ for immigrants and natives, but “doing so is not equivalent to distinguishing between domestic and foreign source human capital in the analysis of earnings determination as this distorts the full picture.”¹⁵

Through her analysis, Friedberg (2000) estimates that returns in terms of earnings to schooling and earnings to experience are 8% and 0.7%, respectively (see appendix, table III). Upon arrival, immigrants are found to earn nearly 25% less than natives. Every year following migration their relative earnings rise by 0.8%. The return to domestic schooling and domestic experience is 8.8% and 1.4%, respectively, while the return to foreign schooling and foreign experience is 7.6% and 0.3%, respectively. While there is an 8% return to domestic schooling for immigrants, it is 10% for natives. Immigrants’ foreign schooling yields a 7.1% return. The results for experience show a similar pattern: “native earnings rise by 1.7% for each year of domestic experience, while immigrants gain 1.1% for each year of domestic experience and just 0.1% for years of foreign experience.”¹⁶ These considerable earning gaps between immigrants and natives can be fully explained by the lower value placed on the immigrants’ human capital, the author, Friedberg (2000), comments. However, she does not offer a qualitative view on the why this may be.

Inspired by Friedberg’s findings and methods, we will present

¹³Goldner et al., 2012

¹⁴Friedberg, 2000

¹⁵Ibid.

¹⁶Ibid.

our general function and our population regression function in section VI. We will now present our *Data and Sources*.

IV. Data and Sources

As mentioned in section II, relevant and reliable data in regards to earnings and employment outcomes of refugees settled in the U.S. is scant. There are some state-by-state case studies, like the one from Idaho cited above, that serve to shed some light on the subject, but for our cross-sectional study of refugees, we gather our data using a certain hypothesis as presented in a paper published by the Migration Policy Institute (MPI) in 2015, titled “The Integration Outcomes Of U.S. Refugees: Successes And Challenge.”

In their research, the MPI (2015) estimated demographic and socioeconomic indicators for the U.S. refugee population resettled between 1980 and 2011, using American Community Survey (ACS) data from the U.S. Census Bureau. The American Community Survey (ACS) is an ongoing statistical survey by the U.S. Census Bureau. It regularly gathers information previously contained only in the long form of the decennial census, such as ancestry, educational attainment, income, language proficiency, migration, disability, employment, and housing characteristics.¹⁷

Since the ACS does not identify refugees separately from other immigrants, the analysis conducted by the MPI (2015) relies on distinguishing refugee status based on characteristics of immigrants available in the ACS: country of birth and year of arrival in the U.S. They then matched the Immigrants’ characteristics from the ACS data against administrative data on refugee admissions from the Department of Homeland Security (DHS) and the Department of State’s Bureau of Population, Refugees, and Migration (PRM) and its Refugee Processing Center’s Worldwide Refugee Admissions Processing System (WRAPS). The DHS and PRM accurately provide the number of refugees arriving by year and country of origin. Refugee sta-

¹⁷ACS, 2013

tus was assigned to every country/year combination in which refugee admissions reported by the DHS and PRM exceeded 40 percent of the estimated foreign-born population identified in the ACS data.

For our research methods general acceptability, conformity, and simplicity, we will consider the ten largest national origin groups arriving during the FY 2002-13 period, as identified by the MPI (2015): Burmese, Iraqis, Somalis, Bhutanese, Cubans, Iranians, Ukrainians, Liberians, Russians, and Vietnamese, to act as a proxy for the refugee population as a whole, for the period of time in question.

We do recognize, and are humbled by, the differences in refugee experiences between each country of origin, but believe that to answer the questions pertinent to the scope of this paper, data from the top ten ‘sending countries’ are satisfactory proxies, since they make up a sizable proportion of the total refugee population. We must also acknowledge, here, that since this method for assigning refugee status to the foreign-born population in the ACS obviously includes refugees and non-refugee immigrants, even after using the forty-percent threshold, there will undoubtedly be some unobserved and unforeseeable biases. We will talk about this, and other limitations in the following sections. Countries from which large non-refugee immigrant inflows likely accompany refugee inflows include Iran, Russia, and Ukraine.¹⁸

The three samples the MPI used for their comparisons were for the years 2009, 2010 and 2011. The core indicators taken directly from the ACS data, are described as: age distribution, English proficiency, educational attainment, employment status, median household income, poverty levels, health insurance coverage, cash welfare receipt, and food stamp receipt. For each indicator, MPI drew comparisons among identified refugees, non-refugee immigrants, and the U.S.-born population as a whole, and among refugees by period of arrival. Since ours is a regression analysis that attempts to find the effects of various independent variables (*regressors*) on a dependent

¹⁸Capps et al, 2015

variable (*regressand*), we will explain our independent variable and dependent variables in the subsequent section. Now, we present our basket of indicators (the appendix contains the relevant IPUMS/ACS codes and their role within each indicator):

1. Total personal income (reports each respondent's total pre-tax personal income or losses from all sources for the previous year)
2. Poverty status¹⁹ (whether an individual falls below the official "poverty line" depends not only on total family income, but also on the size of the family, the number of people in the family who are children, and the age of the head-of-household)
3. Number of family members in household
4. Citizenship (describes the citizenship status of the respondent)
5. Birthplace (indicates the foreign country where the person was born)
6. Ancestry (provides the respondent's self-reported ancestry or ethnic origin)
7. Year of immigration (reports the year in which a foreign-born person entered the United States)
8. English proficiency (measured by varying degrees)
9. Educational attainment (as measured by the highest year of school or degree completed)

¹⁹Poverty, as an indicator, was created using detailed income and family structure information about each individual and by further calculating the family income as a percentage of the appropriate official poverty threshold. For example, if a person's family income is \$25,000 and the poverty threshold for such a person is \$13,861, then the value of POVERTY for that individual is $\$25,000/\$13,861 * 100$ percent, or 180. Individuals whose family income is more than five times the appropriate poverty threshold receive a POVERTY value of 501.

10. Employment status (indicates whether the person is part of the labor force – working or seeking work – and, if so, whether the person was currently unemployed)

In summary, for immigration at least three years prior to 2005, 2006, or 2007 and 2009, 2010, or 2011, our source for primary data is the ACS, and is accessed through IPUMS. Our method for assigning refugee status to a particular set of immigrants follows the same *modus operandi* used by the Migration Policy Institute whereby the status was assigned to every country-year combination in which the ratio of refugee admissions (as reported by the DHS and PRM) to immigrant arrivals was at least 40 percent that year. Our core indicators are: total personal income, poverty status, number of family members in household, birthplace, ancestry, year of immigration, English proficiency, educational attainment, and employment status.

V. Empirical Specifications and Methods

Friedberg (2000) presents the standard earnings function, pioneered by Barry R. Chiswick’s (1978), and conventionally used in the immigration literature as follows:

$$\begin{aligned} \gamma = & \alpha_0 + \alpha_1(ED) + \alpha_2(EXP) + \alpha_3(EXP^2) + \alpha_4(M) \\ & + \alpha_5(YSM) + \alpha_6(YSM^2) + \epsilon \end{aligned}$$

where, γ is the log of annual earnings, ED is years of schooling completed, EXP is years of potential labor market experience, EXP^2 is simply the mathematical square of years of potential labor market experience, M is a dummy variable for immigrant status, and YSM is the number of years elapsed since an immigrant’s arrival in the host country. YSM^2 is simply the mathematical square of the number of years elapsed since an immigrant’s arrival in the host country. ϵ is the error-term or ‘catch-all’. Friedberg (2000) modifies this traditional population function to present the following:

$$\begin{aligned} \gamma = & \beta_0 + \beta_1(M) + \beta_2(ED_1) + \beta_3(ED_2) + \beta_4(ED_2 * M) \\ & + \beta_5(EXP_1) + \beta_6(EXP_2) + \beta_7(EXP_2 * M) + \beta_8(ED_1 * ED_2) \\ & + \beta_9(ED_1 * EXP_2) + \beta_{10}(EXP_1 * EXP_2) + \epsilon \end{aligned}$$

We will paraphrase Friedberg's (2000) explanation of this function but find it imperative to discuss her choice of data first. As is commonplace, the data most widely used to study immigrants in the developed world are presented by the micro-data samples of the U.S. Census Bureau. Friedberg's (2000) qualm is that, "These data do not contain adequate information to reliably determine the source of an immigrant's education, [further], there is no direct measure of where schooling was obtained, and the information on an immigrant's year of arrival (Friedberg, 2000)."

Hence, Friedberg (2000) is persuaded to consider The Census of Population in Israel as being a more appropriate source in her analysis of human capital and its portability. She notes that, "It is possible in this data to precisely date the timing of immigration [and] in addition, the lack of a systematic change in the unobserved quality of successive immigrant cohorts to Israel makes it possible to use a single cross section of data to analyze assimilation rates. The Israeli case provides a large, richly varied pool of immigrants to observe (Friedberg, 2000)." Friedberg finds that Israeli emigrants come from a wide range of countries and have vastly different educational and occupational backgrounds. Going back to Friedberg's (2000) population equation, we will discuss the individual terms:

$$\begin{aligned} \gamma = & \beta_0 + \beta_1(M) + \beta_2(ED_1) + \beta_3(ED_2) + \beta_4(ED_2 * M) \\ & + \beta_5(EXP_1) + \beta_6(EXP_2) + \beta_7(EXP_2 * M) + \beta_8(ED_1 * ED_2) \\ & + \beta_9(ED_1 * EXP_2) + \beta_{10}(EXP_1 * EXP_2) + \epsilon \end{aligned}$$

where, γ is log earnings, ED is years of schooling completed, EXP is years of potential labor market experience, M is a dummy variable for immigrant status. Since Israel describes the specific country of human capital accrual, $ED1$ and

EXP1 denote years of schooling and potential labor market experience obtained in country or origin and *ED2* and *EXP2* describe years of schooling and potential labor market experience obtained in host country. Further, β_2 and β_5 measure the degree to which the human capital that immigrants acquired in their countries of origin is transferable into earnings potential in their destination country i.e., the “portability” of their human capital after entering Israel. Friedberg’s population function is heavily dependent on education and experience, and the interaction between education and experience.

Our population model is inspired by Friedberg’s (2000) analysis on immigrants in Israel, but in our model that analyzes refugee earnings, we will use age, ancestry, assimilation period, birthplace, educational attainment, employment status, English proficiency, sex, and year of immigration as our regressors. Our regressand is total personal annual income (and variations of the same).

Our general function is described by:

$$\frac{\delta y_{i,r}}{\delta x_n} = f^n \{ \text{age, ancestry, assimilation period, birthplace, educational attainment, employment status, English proficiency, sex, year of immigration} \}$$

And our population model is defined as:

$$\begin{aligned} \gamma_i = & \beta_0 + \beta_1(EDUC) + \beta_2(SEX) + \beta_3(AGE) \\ & + \beta_4(EMPSTAT) + \beta_5(SPEAKENG) + \beta_6(REFUGEE) \\ & + \beta_7(EDUC\#REFUGEE) + \beta_8(FEMALE * REFUGEE) \\ & + \beta_9(MALE * REFUGEE) + \beta_{10}(SPEAKING\#REFUGEE) \\ & + \beta_{11}(EMPSTAT\#REFUGEE) + \mu \quad - \quad \text{equation } \emptyset \end{aligned}$$

where,

$$\begin{aligned} \beta_1(EDUC) = & \delta_1(\text{elementary school}) + \delta_2(\text{middle school}) + \\ & \delta_3(\text{grade 09}) + \delta_4(\text{grade 10}) + \delta_5(\text{grade 11}) + \delta_6(\text{grade 12}) + \\ & \delta_7(1 \text{ year college}) + \delta_8(2 \text{ years college}) + \delta_9(3 \text{ years college}) + \\ & \delta_{10}(4 \text{ years college}) + \delta_{11}(5+ \text{ years college}), \text{ and base category} \\ = & (\text{no schooling}) \end{aligned}$$

$\beta_2(\text{SEX}) = \delta_1(\text{female})$, and base category = (male)
 $\beta_4(\text{EMPSTAT}) = \delta_1(\text{unemployed}) + \delta_1(\text{not in labor force})$,
 and base category = (employed)
 $\beta_5(\text{SPEAKENG}) = \delta_1(\text{speaks only English}) + \delta_2(\text{speaks very well}) + \delta_3(\text{speaks well}) + \delta_4(\text{does not speak well})$, and
 base category = (speaks no English)
 $\beta_7(\text{EDUC} \# \text{REFUGEE})$, $\beta_8(\text{FEMALE} * \text{REFUGEE})$,
 $\beta_9(\text{MALE} * \text{REFUGEE})$, $\beta_{10}(\text{SPEAKENG} \# \text{REFUGEE})$,
 and $\beta_{11}(\text{EMPSTAT} \# \text{REFUGEE})$ are our interaction terms
 that measure specific effects of xi on the refugee population.²⁰

The terms in our population model (equation \emptyset) are: γ_i is either total personal income, log of total personal income, or poverty. *EDUC* is educational attainment, *SEX* indicates either female or male, *AGE* represents all ages from 18 to 62, *EMPSTAT* specifies whether the person is part of the labor force or not, *SPEAKING* is the level of English proficiency, *REFUGEE* signals whether the individual is a refugee. The terms that follow *REFUGEE* are our interaction terms, as specified above. And finally, μ is our error term.

We find it incumbent upon ourselves to further explain some variables in our population model, as well as some specific reasons for the particular data selection. Since ours is a regression analysis of the data whereas the MPI's analysis (2015) was simply a comparative one, we find it necessary, to allocate at least 3 years as the time it may take to assimilate into the workforce and reflect in total earnings. There is not an agreed upon average time taken for refugee assimilation in the literature, so we take the liberty of making this hypothesis.²¹ We use the difference between the year of the survey and

²⁰Note: # and \times represent categorical interactions and binary interactions, respectively. Where a binary predictor is a variable that only posses two values: 1 or 0, true or false, refugee or not, etc., a categorical predictor is a variable that can possess a range of qualitative information: level of education (high school completion, college, etc.), English speaking ability (speaks well, does not speak any etc.).

²¹Upon conferring with Prof. Pablo Bose (Associate Professor, Department of Geography, University of Vermont), an expert on refugee

the year of immigration as a tool to realize our assumption.

Our surmise has led to the following: From the American Community Survey 2009-2011 3-Year sample and the American Community Survey 2005-2007 3-Year sample data, we purge all individuals who have been in the country for less than three years.²² We use these 3-year ACS surveys in order to improve the precision of our empirical findings; data in these surveys are collected and combined for 36 months and are controlled to an average of annual Population Estimates over the time period. To be clear, three-year estimates are not an average of the corresponding three one-year estimates, though they may be approximately equal to each other.

The increased precision is best described by the following example: The relationship between the sample sizes used for the estimates means that in many cases one can give an approximate relationship for the SE (Standard Error) or CV (Coefficient of Variance) of one-year and three-year estimates of totals of persons, households, or housing units with certain characteristics. The SE's and the CV's of the three-year estimates are about one over the square root of three, or about 58%, of the one-year estimates.²³

Having now explained the different components in our population model and data set, we are ready to use our model to explain and contrast the earnings of refugees versus the native population. Our aim, through our regressions, is to 'fail to reject the null at a statistically significant level.' Our null hypothesis is $H_0: \mu(\text{earnings for refugee workers}) < \mu(\text{earnings for native born workers})$.

resettlement and author of *Refugees in Vermont* (Bose, Pablo S. 2014. "Refugees in Vermont: mobility and acculturation in a new immigrant destination." *Journal of Transport Geography* 36(2014): 151-159), we find our assumption to be logical.

²²We do this by first generating a variable, *difyear*, and defining it as the difference between the year of the survey and the year of immigration. We then proceed to drop all observations where: *difyear* < 3 and *refugee* == 1. What we have left in the data is refugees who have been given the stated "assimilation period" and all native-born persons.

²³Beaghen and Weidman, 2008

VI. Empirical Findings

Key findings are bolstered by strong validity indicators; the majority of coefficients are found to be statistically significant at the 1% level. There are some outliers that are significant at the 5% and 10%. Our R^2 , or the coefficient of determination, are between 26% and 38% on all our regressions. This is not something that concerns us very much: a lower R^2 may be expected in as the chance of unexplained variability increases with an increase in sample size, and our sample is very large, this does not diminish the fact that our significant coefficients represent a mean change to our regressand in the response to a one unit change to one of our regressors.

A. Our findings for total income, presented in Table A, section IX (appendix), for the base category can be summed up by the following three examples: A 25-year-old (base category is 0 years, but 25 years is more intuitive) native-born male who has no years of schooling, speaks no English, and is currently employed is expected to earn \$20,379 per annum. While a refugee with the same characteristics, this individual is expected to earn \$13,900, which is roughly 30% lower.

Consider a female, 45-years-old, who is a college graduate (4 years of college), speaks English well, and is currently employed. Such a person, if native, would earn an expected \$52,936 annually. A refugee with the same characteristics would earn \$45,587. This amounts to approximately a 14% difference between the two groups. Our third illustration is of an individual who is a 35-year-old male with more than a college degree (a master's or greater, for example). This person is currently employed and speaks English very well. If native, the individual is expected to earn \$92,558 annually. If the person had the same characteristics but was a female refugee, the expected earnings would be: \$77,038, that is, \$15,550 or 16.7% less than the native male.

Using one of the above examples, we will now look at the

log of wages²⁴ and the poverty rate between the two groups where each was a female, 45-years-old, a college graduate (4 years of college), spoke English well, and was employed. From an unconditional mean of 8.68 (from appendix, table A), we would expect earnings to rise by a total of 180%, whereas the refugee with the same characteristics would see her earnings rise by 170% in this log-level model.

The expected poverty level from the third illustration (which expresses total income for the previous year as a percentage of the poverty thresholds) would indicate that a 35-year-old male with more than a college degree (a master's or greater, for example), who is currently employed and speaks English very well, would be 419% above the poverty line for a native male, and 333% above the poverty threshold for a female refugee with the same characteristics.

A few more examples that describe poverty, in a slightly more quantitative manner, are as follows (foot-note 19 provides an example of how poverty is calculated). The first is a native-born female, 30 years of age, has attained eleven years of schooling, speaks English but not well, and is employed. This female would receive a poverty value of 252.318. A refugee with the identical characteristics receives a poverty value of 226.64. A reminder: lower the number, higher the poverty rate.

Another example we may consider is that of a 60-year-old native-born male who has attained five or more years of college education is employed and speaks only English. This male is estimated to receive a poverty value of 498.023. It is interesting to note that this is very close to the 501 value of poverty that puts this individual's family income at more than five times the appropriate poverty threshold. A refugee with these characteristics is estimated to have a poverty value of 430.653.

B. The purpose of presenting Table B, Appendix, is to observe any differences in total income between the refugees

²⁴Log of wages is used as a gauge to measure the percent change in wages as opposed to an absolute change in wages.

and native-born individuals before the Great Recession and during it i.e., in the periods 2005-2007 and 2009-2011, respectively. Let us consider three examples here, too. The first is of an individual who is 45 years old, is male, has completed four years of college, is employed, and speaks English well. In 2007, a native-born individual fitting these characteristics is estimated to earn a total income of \$71,874, whereas, a refugee with the same characteristics would earn \$59,961. In 2011, the two individuals would earn \$73,205 and \$60,821, respectively. Again, we see a discount simply because the individual is a refugee, but we do not see a drastic difference post-crisis, and during-crisis. This may be because of these individuals' educational attainments.

For the second example, let us consider an individual who a female aged 27, who has completed 10 years of schooling, speaks English but not well, and is currently not in the labor force. A native-born female meeting these characteristics in 2007 would earn an estimated total income of \$(-19,750). Her refugee counterpart would earn about \$(-19,973). In 2011, their respective incomes would be \$(-21,116), and \$(-24,993). We see here that pre-crisis, both the females netted about the same negative income. The effect of the crisis may be seen in their income in 2011, where the gap increases by nearly \$3,000, and where both individuals earn less than what they did pre-crisis.

Our last example will involve a male aged 36, who has completed junior high (grade 8), speaks English well and is currently not employed. A native-born male with these characteristics in 2007 would earn an estimated total income of \$10,618. A refugee with the same characteristics would earn about \$5,958. In 2011, their incomes would be \$8,338, and \$3,673, respectively. What we see here is that there is about a \$5,000 discount in being a refugee in both periods, with total incomes lowered for the native-born male and refugee male during the crisis when compared to pre-crisis incomes.

VII. Conclusion

The outcomes are stark but unsurprising. We find that refugees face a penalty on their earnings when compared to native-born individuals with comparable human capital traits; after calculating annual income for the two groups in a variety of permutations and combinations, we find that the lowest spread between total earnings for refugees and native-born workers is roughly 10% and the highest spread is roughly 34%. Our null hypothesis was set up as $H_0: \mu(\text{earnings for refugee workers}) < \mu(\text{earnings for native born workers})$. We fail to reject our null.

It may also be said that refugees are more likely to see their incomes take an adverse hit during a recession when compared to a native-born individual. Added to this, our findings on poverty levels shed light on the fact that refugees are more likely to be closer to the poverty line than their native-born counterparts. These disparities, if Human Capital Theory is held to its original essence, should not exist; the well-being of comparable refugees and native-born Americans should not be very different.

Hence, these findings should give us pause. Is it truly the fact that the human capital accumulated by refugees is inherently less valuable than a native-born American's? Or could it be that the human capital between two identical individuals is perceived through a veil of doubt or discrimination based on a person's origins? It may very well be a combination of these factors but whatever the case may be, further research is to be undertaken on the effects of better housing integration, more vocational and language training opportunities, or even larger financial assistance, on the short-term and long-term earnings of refugees.

Future Research

Potential future research may involve comparing the earnings of refugees to other immigrants. This would be vital in understanding the disparities among these groups. A sensitivity

analysis may be conducted as well to see differences in earning within the same ethnic group where one cohort is refugees and the other is immigrants.

Contingent on improvement in data, an analysis like Freidberg's (2000) may give researchers an even more thorough picture of the transferability of human capital i.e., if researchers in the future have access to the specific location of the accumulated human capital, it may be paint a more holistic picture of the discounts that employers place on foreign human capital accumulation versus domestic.

Limitations

There are limitations to our study, no doubt. These could include our methodology for assigning refugee status to a group of immigrants, the notion that refugees might want to eventually leave the country which may cause employers to view them unfavorably when compared to a permanent resident, the human capital that refugees have accumulated in their home countries truly being of an inferior quality, among others.

Although there are no official figures for the number of refugees that want to return to their homelands and how many end up doing so (in the case of U.S. refugees at least), to address this point, we allow for an 'assimilation period' (see footnote 21, page 19) of at least three years. In doing so, we hope to capture a possible change in sentiment in regards to a new-found identity that the refugees may have developed in their new homes.

Since the American Community Survey, and the census data in general, does not identify refugees separately from immigrants, the method we used to assign refugee status²⁵ may well have led to individuals not considered as refugees but being counted as such. One way we were able to check this problem was to look at snapshots in time for official refugee num-

²⁵Refugee status was assigned to every country/year combination in which refugee admissions reported by the DHS and PRM exceeded 40 percent of the estimated foreign-born population identified in the ACS data.

bers and compare them with refugee assignments that were made with the data we had available.

We found that from the countries in our study, the official DHS numbers were greater than the number of refugees we were able to tag using our methodology. This leads us to postulate that the refugees tallied by the DHS were adequately accounted for within our study. For instance, for the year 2005, the DHS reported a refugee inflow of 33,426 persons. The number of individuals we tagged as refugees was 1,758. Similarly, for 2006 the refugee inflow was 28,994 persons. The number was assigned as refugees via the ACS data was 1,730. Another snapshot: for the years, 1994 and 1995, the number of official arrivals were 56,140 and 48,867, respectively. The number of refugees we were able to tag in our analysis were 4,772 and 5,518, respectively.

It is unfortunate that more robust and comprehensive data about refugees in the United States is unavailable; this was the biggest challenge we faced in our research. Until more data becomes available, various assumptions will have to be made, but under the right conditions and suppositions, as we have hopefully undertaken in this paper, reasonable inferences can be made with the limited data available today. There is a long way to go in terms of achieving income parity between refugee workers and native-born workers. This paper hopes to have shed some valuable light on the *Transferability of Human Capital*.

References

- [1] All Census Bureau and ACS data were accessed from the Integrated Public Use Microdata Series (IPUMS): Steven Ruggles, J. Trent Alexander, Katie Genadek, Ronald Goeken, Matthew B. Schroeder, and Matthew Sobek. Integrated Public Use Microdata Series: Version 5.0 [Machine-readable database] (Minneapolis: University of Minnesota, 2010), <http://usa.ipums.org/usa/>.
- [2] ACS, United States Census Bureau. “Summary File. 2007 – 2011 American Community Survey”. 2013. U.S. Census Bureau’s American Community Survey Office. Web. 1 October, 2016
- [3] Beaghen, M. And Weidman, L. (2008). Statistical Issues Of Interpretation Of The American Community Survey’s One-, Three-, And Five-year Period Estimates. U.S. Census Bureau.
- [4] Becker., Gary S. ”Human Capital.” The Concise Encyclopedia of Economics. 1993. Library of Economics and Liberty. Retrieved February 7, 2018 from the World Wide Web: <http://www.econlib.org/library/Enc1/HumanCapital.html>
- [5] Becker, Gary S. (1964, 1993, 3rd ed.). Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education. Chicago, University of Chicago Press. ISBN 978-0-226-04120-9.
- [6] Becker, Gary S. “Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education.” 1975. 2d ed. New York: Columbia University Press for NBER.
- [7] Bowles, S. and Herbert Gintis. 1975. ”The Problem with Human Capital Theory—A Marxian Critique.” *The American Economic Review* 65.2: 74-82. Web.

- [8] Capps, Randy and Kathleen Newland with Susan Fratzke, Susanna Groves, Gregory Auclair, Michael Fix, and Margie McHugh. (2015). *The Integration Outcomes of U.S. Refugees: Successes and Challenges*. Washington, DC: Migration Policy Institute.
- [9] Chiswick, B. (1978). The Effect of Americanization on the Earnings of Foreign-born Men. *Journal of Political Economy*, 86(5), 897-921. Retrieved from <http://www.jstor.org/stable/1828415>
- [10] Duleep, H. O., & Regets, M. C. (1999). Immigrants and Human-Capital Investment. *American Economic Review*, 89(2), 186-191. doi:10.1257/aer.89.2.186
- [11] Friedberg, R. (2000). You Can't Take It with You? Immigrant Assimilation and the Portability of Human Capital. *Journal of Labor Economics*, 18(2), 221-251. doi:10.1086/209957
- [12] Goldner, S., Eckstein, Z., & Weiss, Y. (2012). *Immigration and Labor Market Mobility in Israel, 1990 to 2009*. MIT Press. Retrieved from <http://www.jstor.org/stable/j.ctt5hhm0m>
- [13] Messick, Molly. (2012, January 26). The Recession and its Effects on Refugee Resettlement. Retrieved February 7, 2018 from <https://stateimpact.npr.org/idaho/2012/01/26/the-recession-and-its-effects-on-refugee-resettlement/>
- [14] McCausland, W. D., Summerfield, F. E., & Theodossiou, I. (2015). "Are Individual Characteristics all that Matters in Earnings Determination? Evidence from the US and Germany." (Discussion Paper in Economics; Vol. 15, No. 14).
- [15] Pope, Amy (2015, November 20). The Screening Process for Refugee Entry into the United States. Retrieved February 7, 2018, from

<https://obamawhitehouse.archives.gov/blog/2015/11/20/infographic-screening-process-refugee-entry-united-states>

- [16] Spence, M. (1973). Job Market Signaling. *The Quarterly Journal of Economics*, 87(3), 355-374. Retrieved from <http://www.jstor.org/stable/1882010>
- [17] Tamborini, Christopher R., ChangHwan Kim, and Arthur Sakamoto. 2015. "Education and Lifetime Earnings in the United States." *Demography* 52: 1383–1407.
- [18] United States Census Bureau (2011). Summary File, 2007 – 2011 American Community Survey. U.S. Census Bureau's American Community Survey Office, 2013. Web. 1 October, 2016.
- [19] Vijaya, R. M., Miller, M., & Fletcher, D. (2017, August 15). Analysis — Within 7 years, refugees are self-sufficient and contributing to the U.S., on average. Retrieved February 07, 2018, from https://www.washingtonpost.com/news/monkey-cage/wp/2017/08/15/within-7-years-refugees-are-self-sufficient-and-contributing-to-the-u-s-on-average/?utm_term=.a097013b32d7

Appendix

Continued from Footnote 1. The screening process for refugee entry into the U.S. is as follows (Pope, 2015):

1. Refugee applicants identify themselves to the U.N. Refugee Agency, UNHCR.
2. Applicants are received by a federally-funded Resettlement Support Center (RSC).
3. U.S. security agencies screen the candidate. These agencies include the National Counterterrorism Center, FBI, Department of Homeland Security, and the State Department.
4. The Department of Homeland Security and the United States Citizenship and Immigration Services interview each candidate.
5. Biometric security checks. If not already halted, this is the end-point for cases with security concerns. Otherwise, the process continues.
6. Medical check: This is the end-point for cases denied due to medical reasons. Refugees may be provided medical treatment for communicable diseases such as tuberculosis.
7. Cultural orientation and assignment to domestic resettlement locations: Resettlement agencies try to place people where there's an existing immigrant community from the same country. Factors like whether there are any special health needs are also taken into consideration. Prior education is not considered when placing a refugee in a particular location. (This is an important point to note, as we will see in our regressions to follow).
8. Travel and arrival to the U.S.
9. Integration: For the first 90 days after refugees arrive, the contracted resettlement agency is responsible for providing them with food, shelter, medical care and other services. They also help them find work, reaching out to local employers. The Office of Refugee Resettlement at the State Health and Human Services provide short-term cash and medical assistance to new arrivals, as well as case management services, English as a Foreign Language classes, and job readiness and employment services – all designed to facilitate refugees' successful transition in the U.S. and help them to attain self-sufficiency.

Table I: Author’s representation of unemployment rate by educational attainment, 2002 – 2017

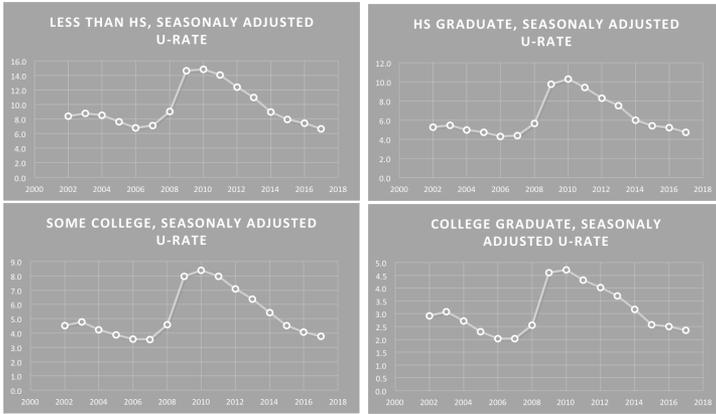
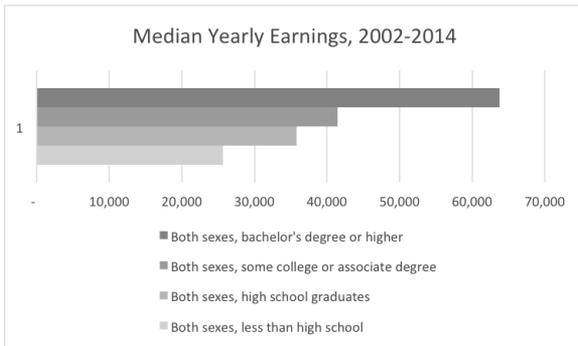


Table IIa: Author’s representation of median yearly earnings, 2002 – 2014



Source: Data from the Bureau of Labor Statistics, accessed January 8, 2018

Table IIb: Author's representation of weekly earnings by educational attainment, 2002 – 2014



Source: Data from the Bureau of Labor Statistics, accessed January 8, 2018

Table III: Author's representation of Friedberg's estimation of return to human capital, measured in the log of monthly earnings.

	(a)	(b)	(c)
Immigrant	-0.253 (.008)	0.04 (.006)	0.374 (.233)
Education	0.081 (.006)		
Education abroad		0.076 (.001)	0.071 (.001)
Education in Israel		0.088 (.001)	0.1 (.001)
Experience	0.0068 (.0002)		
Experience abroad		0.0032 (.0003)	0.001 (.0003)
Experience in Israel		0.014 (.0002)	0.017 (.0004)
Years since migration	0.0081 (.0003)		
Education in Israel \times immigrant			-.020 (.002)
Experience in Israel \times immigrant			-.0057 (.0005)
R^2	0.254	0.266	0.269
N	54103	54103	54103

Note: Standard errors are in parentheses. As explained by Friedberg (2000), the first column in the table above represents the standard specification that constrains the returns to schooling and experience to be invariant to the worker's nativity and to where the human capital was obtained. The second column in the table shows that human capital obtained abroad is of significantly less value in the Israeli labor market than human capital obtained in Israel. The third column illustrates that the return to domestic human capital is higher than the return to foreign human capital, and it is higher for natives than for immigrants.

Table A: Author's presentation of the three regressands as explained by the various regressors

	Total Income	Log (Total Income)	Poverty
<i>Education (base == no education)</i>			
1. nursery school to grade 4	-1,580.078 (6.29)**	-0.029 (2.26)*	-5.732 (3.60)**
2. grades 5, 6, 7, or 8	-1,699.909 (12.74)**	-0.080 (11.83)**	-13.195 (16.04)**
3. grade 9	-467.492 (3.57)**	-0.065 (9.49)**	-6.716 (8.15)**
4. grade 10	-191.818 (1.55)	-0.058 (9.06)**	7.234 (9.18)**
5. grade 11	1,374.969 (11.50)**	-0.195 (31.03)**	44.807 (57.37)**
6. grade 12	4,464.824 (39.27)**	0.207 (36.03)**	78.806 (109.26)**
7. 1 year of college	9,859.909 (84.93)**	0.306 (52.88)**	98.475 (135.19)**
8. 2 or 3 years of college	13,533.079 (113.59)**	0.537 (92.03)**	126.682 (172.07)**
9. 4 years of college	31,489.445 (257.54)**	0.823 (141.99)**	173.656 (239.26)**
10. 5+ year of college	56,772.155 (388.10)**	1.122 (192.33)**	197.761 (271.40)**
<i>Sex (base == male)</i>			
female	-19,596.175 (650.06)**	-0.459 (650.06)**	-7.547 (77.86)**
<i>Age (base == 0 years)</i>			
age (18 to 62)	711.083 (728.40)**	0.029 (977.14)**	2.544 (631.13)**
<i>English Speaking (base == speaks no English)</i>			
1. speaks only English	3,900.074 (9.06)**	0.208 (9.49)**	53.119 (23.10)**
2. speaks very well	3,321.314 (7.66)**	0.191 (8.69)**	37.536 (16.27)**
3. speaks well	1,469.467 (3.28)**	0.124 (5.53)**	28.391 (12.01)**
4. speaks, but not well	1,216.107 (2.50)*	0.081 (3.52)**	44.118 (17.86)**
<i>Employment status (base == employed)</i>			
1. unemployed	-23,899.035 (595.77)**	-1.029 (487.89)**	-98.365 (420.23)**
2. not in labor force	-28,697.930 (1,153.23)**	-1.255 (970.54)**	-108.566 (796.30)**

Table A [continued]

refugee	-16,400.548 (21.79)**	-0.187 (10.57)**	-61.134 (29.86)**
<i>Interaction terms</i>			
base.educ#1.refugee	10,281.404 (13.72)**	0.240 (11.70)**	61.560 (23.78)**
1.educ # 1.refugee	11,296.587 (12.38)**	0.210 (6.14)**	43.592 (10.06)**
2.educ # 1.refugee	10,398.597 (13.95)**	0.240 (12.99)**	66.122 (27.95)**
3.educ # 1.refugee	9,544.031 (12.21)**	0.305 (13.38)**	62.386 (21.84)**
4.educ # 1.refugee	9,614.338 (11.96)**	0.294 (12.66)**	53.171 (17.32)**
5.educ # 1.refugee	7,932.632 (9.18)**	0.316 (11.82)**	20.424 (6.43)**
6.educ # 1.refugee	5,356.883 (8.15)**	0.098 (9.60)**	6.875 (5.29)**
7.educ # 1.refugee	2,817.693 (4.06)**	0.063 (5.17)**	6.306 (4.07)**
8.educ # 1.refugee	2,487.879 (3.47)**	0.012 (0.96)	1.349 (0.80)
9.educ # 1.refugee	-1,628.504 (2.24)*	0.001 (0.04)	-3.337 (2.58)**
10.educ # 1.refugee	(omitted due to collinearity)		
female × refugee	4,796.486 (18.86)**	0.101 (17.13)**	21.285 (26.61)**
male × refugee	(omitted due to collinearity)		
base.speakeng #	-382.814	-0.082	17.035
1.refugee	(0.68)	(3.05)**	(5.61)**
1.speakeng # 1.refugee	16,358.990 (26.78)**	0.302 (20.69)**	59.088 (30.26)**
2.speakeng # 1.refugee	15,680.610 (42.45)**	0.325 (28.36)**	82.967 (53.94)**
3.speakeng # 1.refugee	5,884.612 (16.28)**	0.194 (16.11)**	55.997 (34.65)**
4.speaking # 1.refugee	(omitted due to collinearity)		
base.employment #	21.861	-0.161	-23.274
1.refugee	(0.10)	(13.52)**	(21.15)**
1.employment #	-4,507.066	-0.102	-19.818
1.refugee	(13.31)**	(4.93)**	(9.99)**
2.employment #	(omitted due to collinearity)		
1.refugee	(omitted due to collinearity)		
constant	7,579.400 (17.15)**	8.686 (386.07)**	94.743 (39.62)**
R^2	0.28	0.38	0.27
N	9,144,714	8,230,998	9,144,714
* p < 0.05; ** p < 0.01			

Table B: Author's presentation of total income pre-crisis and during-crisis

	2005-2007	2009-2011
	Total Income	Total Income
<i>Education (base == no education)</i>	0	0
	(.)	(.)
1. nursery school to grade 4	-2301.1*** (-5.88)	-1420.7*** (-4.34)
2. grade 5, 6, 7, or 8	-3141.6*** (-13.99)	-736.1*** (-4.32)
3. grade 9	-1583.7*** (-7.11)	156.5 (0.95)
4. grade 10	-1286.3*** (-6.00)	406.3** (2.64)
5. grade 11	-98.94 (-0.47)	2326.0*** (15.91)
6. grade 12	3870.7*** (19.06)	4553.8*** (33.20)
7. 1 year of college	9426.5*** (45.73)	9807.4*** (69.72)
8. 2 years of college	12721.9*** (60.76)	13784.5*** (94.49)
10. 4 years of college	30403.2*** (142.83)	31999.3*** (211.80)
11. 5+ years of college	54253.1*** (225.20)	58668.5*** (309.76)
<i>Sex (base == male)</i>	0	0
	(.)	(.)
2.female	-20346.6*** (-475.90)	-18957.1*** (-445.68)
Age	697.5*** (490.95)	722.1*** (537.03)
<i>English Speaking (base == speaks no English)</i>	0	0
	(.)	(.)
3. yes, speaks only English	4014.5*** (6.55)	3817.0*** (6.35)
4. yes, speaks very well	3450.0*** (5.59)	3201.4*** (5.29)
5. yes, speaks well	1603.6* (2.51)	1365.9* (2.18)
6. yes, but not well	1516.1* (2.20)	930.0 (1.36)
<i>Employment status (base == employed)</i>	0	0
	(.)	(.)
2. unemployed	-21432.8*** (-330.72)	-25633.0*** (-496.40)
3. not in labor force	-26944.8*** (-743.49)	-30337.9*** (-884.94)

Table B [continued]

refugee	-15681.2*** (-14.67)	-17332.3*** (-16.37)
<i>Interaction terms</i>		
0.educ#1.refugee	12584.9*** (10.74)	8794.5*** (8.69)
1.educ#1.refugee	12441.8*** (9.14)	10548.9*** (8.53)
2.educ#1.refugee	11506.0*** (10.97)	9403.6*** (8.90)
3.educ#1.refugee	10279.8*** (9.64)	8845.4*** (7.84)
4.educ#1.refugee	11122.4*** (9.57)	8328.3*** (7.47)
5.educ#1.refugee	9973.6*** (6.89)	6405.3*** (5.90)
6.educ#1.refugee	6500.7*** (7.01)	4371.4*** (4.70)
7.educ#1.refugee	3649.5*** (3.68)	2219.0* (2.28)
8.educ#1.refugee	3713.8*** (3.63)	1416.0 (1.41)
10.educ#1.refugee	-1160.0 (-1.13)	-1984.8 (-1.94)
11.educ#1.refugee	0 (.)	0 (.)
Female × refugee	4335.4*** (11.56)	5127.8*** (14.84)
Male × refugee	0 (.)	0 (.)
1.speakeng#1.refugee	-619.5 (-0.76)	-153.0 (-0.20)
3.speakeng#1.refugee	16988.5*** (18.39)	15894.0*** (19.50)
4.speakeng#1.refugee	16137.2*** (30.43)	15301.1*** (29.63)
5.speakeng#1.refugee	6378.1*** (12.29)	5438.7*** (10.77)
6.speakeng#1.refugee	0 (.)	0 (.)
1.employed #1.refugee	-1450.0*** (-4.48)	1494.3*** (5.39)
2.not employed #1.refugee	-6863.4*** (-10.85)	-2173.8*** (-5.39)
3.not in labor force #1.refugee	0 (.)	0 (.)
<i>Constant</i>	8479.5*** (13.30)	7346.0*** (11.98)
N	4506001	4638713
R ²	0.2626	0.2920

* p < 0.05; ** p < 0.01

Table C: An illustration of the addition of coefficients on interaction terms in a simplified model

$$\text{Total Income} = \beta_0 + \beta_1 \text{college} + \beta_3 \text{refugee} + \beta_4 \text{college} \times \text{refugee}$$

	Native	Refugee
Non-College	β_0	$\beta_0 + \beta_2$
College	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$

A Sequential Game-Theoretic Approach to Student Learning Time

Luther Yap

Abstract

Time can be formulated as a production input in a student's learning, and the time that a student spends in the classroom and studying alone can have differing productivities. This paper presents a sequential game-theoretic approach to student learning time, showing how an administrator's choice of time bounds can subsequently influence the teacher, parent, and the student time allocations. The model suggests that low returns of additional lesson time can be explained by a teacher's low productivity and responsiveness, and the relatively high productivity of a student's out-of-school learning time. These results are robust to some alternative formulations such as omitting a player, endogenous productivity, and imperfect information.

I. Introduction

There has been considerable interest recently about the effect of increasing lesson time on learning for students receiving primary and secondary education. Among others, Gromada and Shewbridge (2016) did a literature review of student learning time and developed a qualitative model of using allocated instruction time in an OECD education working paper; Lavy (2015) used Program for International Student Assessment (PISA) data to analyse the relationship between lesson time and test performance. A student's learning is typically measured by his test scores, which allows empirical studies in this field. In many studies, there is a small but significant positive effect of lesson time on test scores. (Cattaneo, Oggenfuss and Wolter, 2016; Lavy, 2015; Rivkin and Schiman, 2015) Other studies suggest no significant relationship. (Woessmann, 2010; Pischke, 2007)

In view of existing empirical studies, this paper aims to present a model to explain the phenomenon where additional

lesson time has either a small effect or no effect on test scores. Time is typically viewed as a key resource in the production of learning in many theoretical papers (Brown and Saks, 1984; Levin and Tsang, 1987). In the education literature, it is well-recognised that learning requires time (Stoll et al, 2003), and time is a scarce resource that needs to be allocated. (Lazear, 2001) Thus, the allocation of a student's time to maximise learning (Y) among other objectives (Z) is the central problem of this model. This model will contribute to existing debates on lesson time by providing a game-theoretic explanation of how parents and students actively respond to lesson time, thereby complementing conventional explanations on teacher effectiveness in the education literature.

This theoretical paper will model the allocation of student time as a sequential game with perfect information involving four players. The administrator (player 1) chooses an upper and lower bound for the teacher (player 2) to allocate lesson time. Based on the teacher's choice of lesson time, the parent (player 3) has a fraction of the student's total time left, and she chooses the amount of free time left for the student.¹ The student (player 4) then allocates his remaining time. The subgame perfect Nash equilibrium of the game will be found using backward induction.

The main research questions that this model aims to answer are:

1. Why does an increase in lesson time have a small effect on student learning?
2. How can learning outcomes be improved?

The first question refers to the model's account of the existing empirical phenomenon from a theoretical perspective. The second question is an application of the model: since the

¹The parent would have considerable autonomy over the student's remaining time choice for students in primary and secondary education. This paper will refer to "students" as primary and secondary students, although some of the results would also be analogously relevant to tertiary students.

model shows the relationship between time and learning, its parameters can inform policy-makers on how learning can be improved. The thesis of this paper is that the sequential game-theoretic model explains low returns of additional lesson time through a teacher's low productivity and responsiveness, and the relatively high productivity of a student's out-of-school learning time.

In the rest of the paper, Chapter 2 does a brief literature review of empirical and theoretical work done in this field. Chapter 3 expounds on and solves the sequential game-theoretic model. Chapter 4 explains how the model answers the two main research questions. Chapter 5 will address possible objections to the argument by using alternative formulations in response to dropping some assumptions. Chapter 6 discusses the model's limitations and potential for further research.

II. Literature Review

Recent work that elucidates the relationship between learning time and test scores is largely empirical. Lavy (2015) used fixed effects estimation for 2006 PISA data to account for student heterogeneity, taking advantage of within-student variation in test scores. Rivkin and Schiman (2015) used a similar approach with PISA 2009, adding controls for school quality. Their results are largely similar, suggesting a small but positive and significant relationship. Cattaneo, Oggenfuss and Wolter (2016) used a similar approach but applied it to Switzerland's data instead, also yielding similar results. Grave (2011) found, among tertiary students, that time devoted to lesson and to self-study were both positively correlated with test scores. Besides econometric approaches, there are also meta-analytic reviews (Kindron and Lindsay, 2014) and literature reviews (Gromada and Shewbridge, 2016).

This paper stems more from theoretical work that aims to explain incentive structures surrounding student learning. The microeconomics of schooling took off after the Brown and Saks' (1981) paper that looks at economic decisions that the teacher has to make in the classroom, characterised by optimi-

sation under constraint. They applied microeconomic models to teachers' choice problems, such as when the teacher has to choose among several lesson materials, and when the teacher has to maximise overall learning when there are many students. Subsequently, Levin and Tsang (1987) formulated the student time allocation as an optimisation problem, where the student has to allocate a limited amount of time and effort between in-school and out-of-school activities.

Thereafter, Lazear (2001) modelled the education production function with class size as one of the arguments, thereby analysing factors that lead to education production, including how well-behaved the students are, and the influence of segregating students by type. Coates (2003) looked at instructional time as a production input in education that could interact with class size and teacher quality, verifying this model with a study in Illinois. Todd and Wolpin (2003) looked at the family's choice problem in deciding its input for the child's education and the school that the child is placed in. Schalock, Cowart and Staebler (1993) argued that the productivity of lesson time is a function of many factors that can be classified under teacher variables, student variables, school variables, and community variables through a complicated network. Their work was largely qualitative.

This game-theoretic model aims to combine the individual theoretical models into a coherent framework, highlighting the role of time allocation. Levin and Tsang looked at student choice; Todd and Wolpin at family choice; Brown and Saks at teacher choice. This model would put these players into a sequential game, modelled partially by Schalock et al (1993) as they presented a network with four groups of variables: the school/administrator, the teacher, the community/parent, and the student. By using a game-theoretic optimisation problem, this paper also formalises their idea.

III. A Game-Theoretic Model

A. Conceptual Framework

The allocation of student learning time can be modelled as a sequential game involving four players – an administrator (A), a teacher (R), a parent (P), and a student (S). In the exposition of the model, we will look at the case of one administrator, one teacher, one parent, and one student, but the results can easily be generalised. In this section, the order of moves will be discussed before the incentive structure of each economic agent and a functional form for learning.

Order of Moves

The order of moves is as such:

1. The administrator (A) chooses a minimum and maximum allocated time for teaching, such that the teacher's choice must be between these lower and upper bounds. We denote the minimum allocated time as t_A and the maximum as $t_A + \mu$, so μ is the flexibility a teacher has in choosing lesson time.
2. The teacher (R) chooses an instruction time for the student (t_{R1}), where $t_A \leq t_{R1} \leq t_A + \mu$. In doing so, the teacher indirectly chooses the amount of time left that the parent can allocate for the student (T_P). Where T is the total amount of time available for the student, $T = T_P + t_{R1}$.
3. The parent (P) chooses how the student's remaining time after lessons, T_P should be allocated between learning enforced by the parent (t_{P1}), non-academic activities (t_{P2}), and the student's free time (T_s) i.e., $T_P = t_{P1} + t_{P2} + T_s$.
4. The student (S) then allocates her remaining time between academic and non-academic activities.

Since this model focuses on the relationship between time and learning, and all players want the student to learn, the

utility function of any agent i will be $U_i(Y, Z_i)$, where Y is the student's learning,² and Z_i is a variable that catches everything else that the agent might be interested in optimising. The utility function is strictly increasing in both arguments i.e., $\frac{\delta U_i}{\delta Y}, \frac{\delta U_i}{\delta Z_i} > 0$. It is assumed that the utility function is quasi-concave so that a unique critical point will be the maximum. For simplicity, we will use the Cobb-Douglas utility function in this analysis, so β_i weighs agent i 's other objective against the student's learning Y .³

$$U_i(Y, Z_i) = Y Z_i^{\beta_i} \quad (1)$$

Incentive Structures

The administrator aims to maximise his objective function subject to a budget constraint (B). This theoretical administrator may function in several capacities. In a government, this budget constraint could be the allocation of funds between the education sector and other national needs. In a school, this budget constraint could be a competition of funds between teacher salary and other infrastructural needs. When the allocated teaching hours increases, teachers have to be compensated, so we will take the compensation per hour of t_A as w_1 . When the upper bound increases, teachers would also have to be compensated, so the compensation per hour of μ is w_2 . The cost per unit of Z_A is w_3 . Consequently, the budget constraint can be written as $w_1 t_A + w_2 \mu + w_3 Z_A = B$.

Further explanation is required of variable μ . Suppose that on the same school day, there are two schools with different schedules. The first school has 8 hours of compulsory classes, so students are in their classrooms for the whole day; the second school has 4 hours of compulsory classes, so students would have more flexibility to engage in co-curricular activities in the

²The model refers to Y as learning in general, and is not specific to test scores, but in most empirical studies, the best proxy for learning is test score. This paper will use refer to Y as learning and test score interchangeably.

³It is recognised that any Cobb-Douglas function $F(x, y) = Ax^a y^b$, with $A, a, b > 0$ is quasi-concave.

afternoon. The first school has a high t_A and low μ for that school day, since the teacher has many hours of allocated time, but little flexibility. The second school has a low t_A and high μ , due to flexibility in the afternoon for the teacher to conduct remedial or supplementary classes for students if she so chooses. On that day, total time $t_A + \mu$ would be similar for both schools, and the teachers may expect to be compensated by the same amount, though it is more likely that $w_2 < w_1$. Other ways that schools could increase μ would be through optional summer school programmes or weekend classes. Alternatively, μ can decrease when the school mandates that the students leave the compound by a particular time.

The teacher will maximise her objective function subject to the following constraints:

1. $t_A \leq t_{R1} \leq t_A + \mu$
2. $t_{R1} + t_{R2} = T_R$
3. $t_{R1} + T_P = T$

The first constraint is that teaching time must be between the upper and lower bounds set by the administrator. The second constraint is the teacher's time constraint: she has a limited amount of time T_R , so she has to allocate them between teaching the student t_{R1} and other activities t_{R2} . The third constraint states that a student's total time (T) is allocated between the interaction with the teacher through t_{R1} and the amount of time the parent has left to allocate T_P (which is endogenous in the teacher's problem). T is exogenous, so this means that the chosen t_{R1} cannot be higher than T , since $T_P > 0$ i.e., $t_{R1} < T$. This is trivial as the first constraint already implies $t_{R1} < T$, since $t_A + \mu < T$. The administrator would not select $t_A + \mu < T$, since that would just imply additional cost without additional hours taught. Thus, the teacher's optimisation problem effectively contains two constraints.

Since it is assumed that we are working with a sufficiently young student, the parent allocates the student's remaining time i.e., $t_{P1} + t_{P2} + T_S = T_P$. We can interpret t_{P1} as enforced time spent on academic activities: this could be a parent ensuring that the child does his homework, sending the child for supplementary tuition classes, or personally coaching the child. Then, t_{P2} is the enforced time spent on non-academic activities that would give the parent utility, which might include visiting relatives, or getting the child to do the dishes. The remaining time is then left to the student, and we assume that T_S is non-zero, so every student will still have a minimum amount of time to allocate. In this model, we assume $T_S \geq 2\epsilon$.

The student then allocates his remaining time between academic and non-academic activities. t_{S1} could be additional readings, doing homework on his own accord, etc. and t_{S2} could be playing computer games, resting, co-curricular activities, or other activities that gives him utility. Thus, $t_{S1} + t_{S2} = T_S$. This model also assumes that the student will spend a minimum amount of time on each activity i.e., $t_{S1}, t_{S2} \geq \epsilon$.

There is value in using a weak inequality $t_{S1}, t_{S2} \geq \epsilon$ for the student instead of a strict inequality of $t_{S1}, t_{S2} > \epsilon$. The weak inequality allows us to introduce a parameter ϵ that is exogenous to the student's preferences. This would allow the model to account for "obligatory" time allocations that the student need not necessarily prefer. For instance, in societies where students have a lot of homework and feel huge pressure to succeed academically, students may feel obliged to spend a minimum ϵ time on academic activities even if she prefers otherwise. Conversely, when students are in societies that emphasise all-rounded education, a student might feel obliged to play with his friends thereby spending a minimum ϵ on non-academic activities even if he prefers otherwise. A proper treatment of this term would require two constraints for work and play i.e., ϵ_W, ϵ_P , but for simplicity, this paper will use $\epsilon_W = \epsilon_P = \epsilon$, noting that the conclusions will be the same even if $\epsilon_W \neq \epsilon_P$.

The problems for the four players can be summarised as such:

1. Administrator: $\max_{t_A, \mu, Z_A} U_A(Y, Z_A)$ s.t. $w_1 t_A + w_2 \mu + w_3 Z_A = B$
2. Teacher: $\max_{t_{R1}, t_{R2}} U_R(Y, Z_R)$ s.t. $t_A \leq t_{R1} \leq t_A + \mu$ and $t_{R1} + t_{R2} = T_R$
3. Parent: $\max_{t_{P1}, t_{P2}, T_S} U_P(Y, Z_P)$ s.t. $t_{P1} + t_{P2} + T_S = T_P$ and $T_S \geq 2\varepsilon$
4. Student: $\max_{t_{S1}, t_{S2}} U_S(Y, Z_S)$ s.t. $t_{S1} + t_{S2} = T_S$ and $t_{S1}, t_{S2} \geq \varepsilon$

Where the utility functions are given by equation (1).

Functional Form for Learning

Since the model aims to elucidate the relationship between learning time and overall learning Y , we require Y to be a function of learning time i.e., $Y = Y(t_{S1}, t_{P1}, t_{R1})$. For simplicity, we will use the Cobb-Douglas form for learning:

$$Y = A t_{S1}^{\alpha_1} t_{P1}^{\alpha_2} t_{R1}^{\alpha_3} \quad (2)$$

Several features of equation (2) are notable. Learning time alone, with parents, and with teachers complement each other, so they have a multiplicative relationship. The efficiency of player i in helping the student learn is given by α_1 for the student, α_2 for the parent, and α_3 for the teacher.⁴ The parameter A refers to other factors related to Y that are independent of time, and this will be discussed in the student's problem. With this functional form, it is clear that we require $t_i > 0 \forall i$ strictly.

The Cobb-Douglas functional form deserves further attention. Student learning is a public good because it feeds into the

⁴This paper will use the terms "efficiency" or "productivity" interchangeably to describe α_i parameters. More accurately, α_i is the elasticity of learning with respect to input t_i . For instance, $\frac{\delta \ln Y}{\delta \ln t_{S1}} = \alpha_1$.

utility function of all four players. Education literature has already established how the different players are connected and are instrumental in the student's learning. (Schalock et al, 1993) There is also likely a complementary effect: a teacher's additional input is likely higher for a student if he has done his homework (either by t_{S1} or t_{P1}) than if he has not done his homework, since the student is more likely to understand the teacher's subsequent lesson in the former case.

The equilibrium concept used here will be the Subgame Perfect Nash Equilibrium, where all players are playing mutual best responses in all subgames. To solve this sequential game, backward induction will be used, so the rest of this chapter will solve the respective player's problems starting from the last player. The last section of this chapter will discuss the effect of increasing allocated lesson time based on the model.

B. Student's Problem

The economics of student time has been closely studied by Levin and Tsang (1987). Combining Carroll's learning model (1963) that expressed the degree of learning as a ratio of the time devoted to learning and time needed to learn, and Bloom's time-based curriculum (1968), Levin and Tsang proposed that achievement is a function of four arguments: (1) C , the student's capacity to learn (2) e , effort (3) t , time, and (4) S , level of learning resources both in the school and at home. They treated C and S as being exogenously determined, and the student chooses e and t to maximise her utility. The objective function used was $U = p_1F(Q_1) + p_2G(Q_2)$, where Q_1 and Q_2 are the level of activity in and out of school respectively, and $Q_1 = f(C, S)e_1^{\alpha_1}t_1^{\beta_1}$. This was subject to two constraints: $t_1 + t_2 \leq T$ and $e_1t_1 + e_2t_2 \leq E$. The first-order conditions were then taken and comparative statics done.

Our game-theoretic model differs from Levin and Tsang in several critical ways. Firstly, effort is treated as exogenous and is consequently unbounded. Furthermore, in a four-player game, endogenising student effort will result in the

model becoming very complicated quickly. Thus, a possible formulation of exogenous parameter A in equation (2) can be $A = A(C, e, S)$, where A is strictly increasing in C , e and S . Secondly, while the production of learning is similar to Levin and Tsang in using a Cobb-Douglas function, the student's utility is different. Levin and Tsang used a strongly additive utility function, but our game-theoretic model uses a Cobb-Douglas function, as students can imaginably prefer a variety in academic and non-academic activities.

The student's problem is: $\max_{t_{S1}, t_{S2}} U_S(Y, Z_S) = Y Z_S^{\beta_S} = A t_{S1}^{\alpha_1} t_{P1}^{\alpha_2} t_{R1}^{\alpha_3} Z_S^{\beta_S}$.

Suppose Z_S has a positive linear relationship with time spent on it t_{S2} . We can take the natural logarithm of the equation and simplify the objective function: $\max_{t_{S1}, t_{S2}} \ln A + \alpha_1 \ln t_{S1} + \alpha_2 \ln t_{P1} + \alpha_3 \ln t_{R1} + \beta_S \ln t_{S2}$. This is subject to two constraints: $t_{S1} + t_{S2} = T_S$ and $t_{S1}, t_{S2} \geq \varepsilon$

Forming the Lagrangian for constrained optimisation and taking the first-order conditions, it is straightforward to show that the interior solutions are:

$$t_{S1} = \frac{\alpha_1}{\alpha_1 + \beta_S} T_S \quad \text{and} \quad t_{S2} = \frac{\beta_S}{\alpha_1 + \beta_S} T_S \quad (3)$$

This means that the proportion of time that the student spends on academic and non-academic activities will depend on the ratio of the efficiency of converting time spent into results (α_1) to the student's valuation of non-academic activities (β_S). This case assumes that the second constraint is not binding.

When the second constraint binds for at least one t_{Sj} , we will have three more cases to consider. The solution for two of them are given by:

$$t_{S1} = T_S - \varepsilon \quad \text{and} \quad t_{S2} = \varepsilon \quad (4)$$

$$t_{S1} = \varepsilon \quad \text{and} \quad t_{S2} = T_S - \varepsilon \quad (5)$$

When both t_{S1} and t_{S2} are binding, the solution will be $t_{S1} = t_{S2} = \varepsilon$, which also implies that the parent gave the student the minimum time possible i.e., $T_S = 2\varepsilon$. Notably, in this backward induction procedure, the student's choice t_{S2} is inconsequential for subsequent analysis, since only t_{S1} feeds into the other players' utility function. Thus, the case where the student is given minimum time ($T_S = 2\varepsilon$) is treated similarly to equation (5). There are then three distinct student profiles given by the following definition.

Definition 1. (Strong, average, and weak students).

- *A strong student will choose $t_{S1} = T_S - \varepsilon$*
- *An average student will choose $t_{S1} = \frac{\alpha_1}{\alpha_1 + \beta_S} T_S$*
- *A weak student will choose $t_{S1} = \varepsilon$, where $\varepsilon < \frac{\alpha_1}{\alpha_1 + \beta_S} T_S < T_S - \varepsilon$.*

From this definition, the solution for the strong student corresponds to equation (4), the average student to equation (3), and the weak student to equation (5). It is obvious that an average student will have an interior solution. A strong student will spend as much time as possible on academic activities, given by $T_S - \varepsilon$, while spending the minimum ε on non-academic activities. This likely occurs when she is very efficient in converting time spent on studying into results, so α_1 is high. She also does not value non-academic activities as much as studying, so β_S is low, suggesting a highly motivated learner. A more accurate formulation might be that $\alpha_1 = \alpha_1(C, e, S)$, so capacity (C), effort (e) and resources (S) would influence a student's efficiency in studying. Since these factors are exogenous, using the alternative formulation would not change the results. The weak student is the opposite of

a strong student, so he likely values non-academic activities highly (β_S is high) and is inefficient in converting studying time into learning (α_1 is low), so he will spend the minimum time possible on academic activities, given by $t_{S1} = \varepsilon$. It should be noted that such definitions are formal and need not correspond to the intuitive understanding of strong, average or weak.

C. Parent's Problem

Based on knowledge of the student's best response, the parent will solve a constrained optimisation problem, and the result is summarised in Lemma 1. A detailed proof is written in the Appendix.

Lemma 1. *The parent has a unique optimal solution to each of the three student profiles.*

- *For the strong student:* $t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$,
 $t_{P2} = \frac{\beta_P}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$, $T_S = \varepsilon + \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$
- *For the average student:* $t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} T_P$,
 $t_{P2} = \frac{\beta_P}{\alpha_1 + \alpha_2 + \beta_P} T_P$, $T_S = \varepsilon + \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} T_P$
- *For the weak student:* $t_{P1} = \frac{\alpha_2}{\alpha_2 + \beta_P} (T_P - 2\varepsilon)$,
 $t_{P2} = \frac{\beta_P}{\alpha_2 + \beta_P} (T_P - 2\varepsilon)$, $T_S = 2\varepsilon$

The amount of time the parent gives the student is dependent on α_1 relative to α_2 and β_P . In the cases of the strong and average students, T_S increases when the student is more productive (higher α_1), when the parent values non-academic activities less (lower β_P), or when the parent is less productive in helping the student learn (lower α_2).

It is also unsurprising that the parent allocates more time to a strong student than an average student. Knowing that the strong student will maximise her personal learning time, the parent will give the student more time. This is easily shown:

$$T_S(\text{strong}) = \varepsilon + \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon) =$$

$$\varepsilon + \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} T_P - \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} \varepsilon =$$

$$\frac{\alpha_2 + \beta_P}{\alpha_1 + \alpha_2 + \beta_P} \varepsilon + \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} T_P > \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} T_P = T_S(\text{average})$$

Thus, the allocation of time is given by:

$$T_S(\text{strong}) > T_S(\text{average}) > T_S(\text{weak}) = 2\varepsilon.$$

Proposition 1. *The parent can crowd out a student who is potentially strong or average such that the student becomes weak.*

Crowding out refers to the phenomenon where, with a limited resource T_P , the parent invests heavily in t_{P1} and t_{P2} such that the time left for the student T_S is reduced significantly. Proposition 1 is an interesting phenomenon revealed by the model, and is a corollary of the proof in Lemma 1 in the Appendix. The intuition can be explained with the case of the strong student, since the ability for a strong student to be crowded out implies that the average student can be crowded out, since $T_S(\text{strong}) > T_S(\text{average})$.

Suppose a student is potentially strong, so $t_{S1} = T_S - \varepsilon$, and $T_S = \varepsilon + \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$ according to Lemma 1. For $T_S > 2\varepsilon$, we require $\frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon) > \varepsilon$. However, for sufficiently high α_2 and β_P , it is possible that $\frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon) < \varepsilon$, so the student time constraint is binding and $T_S = 2\varepsilon$, and the weak student outcome will be obtained, where $t_{S1} = \varepsilon$. It is notable that in this case $T_S - \varepsilon = \varepsilon$, but the Definition 1 requires $T_S - \varepsilon > \varepsilon$, so it is still a weak student outcome. Thus, the potentially “strong” student, even when optimising, will display the characteristics of a weak student.

It should be clarified that Proposition 1 suggests that a parent’s choice can make a strong or average student weak, but it does not suggest that parent’s choice can make a weak student average or strong. A high α_2 can occur when the parent is highly intelligent such that her ability to help the student learn far exceeds the student’s own learning ability. A

high β_P can occur when the parent values the student's non-academic activity very highly (as when the child has to help the parent earn a living). Thus, while high α_2 and β_P could be what the parent believes is good for the student, getting the student involved in these activities and leaving very little time left for herself such that $T_S = 2\varepsilon$ can result in a potentially strong student becoming a weak student.

D. Teacher's Problem

With perfect knowledge of the parent and student best response structures, the teacher's optimisation problem can be written as:

$$\begin{aligned} \max_{t_{R1}, t_{R2}} & \ln A + \alpha_1 \ln t_{S1} + \alpha_2 \ln t_{P1} + \alpha_3 \ln t_{R1} + \beta_R \ln t_{R2} \\ \text{s.t. } & t_A \leq t_{R1} \leq t_A + \mu \text{ and } t_{R1} + t_{R2} = T_R \end{aligned}$$

In light of the parent's and the student's problem, it is notable that there is an additional constraint that $T = t_{R1} + T_P$ and $T_P > 2\varepsilon$, noting that the latter inequality is strict. However, this constraint clearly cannot be binding: it is rewritten as $t_{R1} < T - 2\varepsilon$, which will hold when $t_{R1} \leq t_A + \mu$, since the administrator is strictly better off not paying the teacher for hours that will never be used i.e., $t_{R1} \leq t_A + \mu \leq T - 2\varepsilon$.

Lemma 2. *Assuming exogeneity of t_A and μ , for all student profiles, there are three possible solutions to the teacher's optimal choice of instruction time t_{R1} : an interior solution, the lower bound t_A , and the upper bound $t_A + \mu$.*

The detailed proof of Lemma 2 is in the Appendix. The three interior solutions are given by:

$$\frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} + \frac{\beta_R}{T - t_{R1}} = \frac{\alpha_3}{t_{R1}} \quad (6A)$$

$$\frac{\alpha_1 + \alpha_2}{T - t_{R1}} + \frac{\beta_R}{T - t_{R1}} = \frac{\alpha_3}{t_{R1}} \quad (6B)$$

$$\frac{\alpha_2}{T - t_{R1} - 2\varepsilon} + \frac{\beta_R}{T - t_{R1}} = \frac{\alpha_3}{t_{R1}} \quad (6C)$$

Solution (6A) corresponds to the case of the strong student; solution (6B) to the average student; and solution (6C) to the weak student. The equations can be solved analytically to express t_{R1} in terms of other parameters using a quadratic equation, but the expression will be unnecessarily complicated and not particularly meaningful. For comparative statics, equations (6A) to (6C) would suffice. The form of each equation is very similar except for the first term. Since the weak student chooses $t_{S1} = \varepsilon$, the parameter α_1 is inconsequential and does not feature in the teacher's choice. The denominators of the first term follow the parent's choice. In light of their similarity, comparative statics will be detailed for equation (6A) and the same argument for the other two profiles would follow by analogy.

From equation (6A), we can express the parameters in terms of t_{R1} easily.

$$\alpha_1 = (T - t_{R1} - \varepsilon) \left(\frac{\alpha_3}{t_{R1}} - \frac{\beta_R}{T_R - t_{R1}} - \frac{\alpha_2}{T - t_{R1} - \varepsilon} \right)$$

$$\alpha_2 = (T - t_{R1} - \varepsilon) \left(\frac{\alpha_3}{t_{R1}} - \frac{\beta_R}{T_R - t_{R1}} - \frac{\alpha_1}{T - t_{R1} - \varepsilon} \right)$$

$$\alpha_3 = t_{R1} \left(\frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} + \frac{\beta_R}{T_R - t_{R1}} \right)$$

$$\beta_R = (T_R - t_{R1}) \left(\frac{\alpha_3}{t_{R1}} - \frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} \right)$$

From the above equations, we can take the partial derivative of each parameter with respect to t_{R1} . Noting that $\frac{dy}{dx} > 0$ iff $\frac{dx}{dy} > 0$, and $\frac{dy}{dx} < 0$ iff $\frac{dx}{dy} < 0$, we can show how t_{R1} varies with the parameters.⁵ In particular:

$$\frac{\delta t_{R1}}{\delta \alpha_1}, \frac{\delta t_{R1}}{\delta \alpha_2}, \frac{\delta t_{R1}}{\delta \beta_R} < 0 \text{ and } \frac{\delta t_{R1}}{\delta \alpha_3} > 0$$

⁵This property assumes that the inverse function exists. This is not an issue with our existing formulation, and when we restrict the domain of t_i to \mathbb{R}_+^n .

These solutions suggest that the lower bound t_A will be reached for high α_1 , α_2 , β_R and low α_3 . When the parent and student are very competent in converting learning time into results (high α_1 and α_2), and the teacher is less able than they are to do so (low α_3), the teacher would prefer to teach less and give the parent more time to allocate to maximise her own utility. Furthermore, when the teacher cares about things other than instructional time with the student (high β_R), the teacher also has a lower optimal instructional time. The upper bound $t_A + \mu$ will be obtained for the opposite case, and an interior solution otherwise. By analogy, the three possible solutions would be similarly possible for equations (6B) and (6C).

E. Administrator's Problem

With knowledge of the teacher's best responses, the administrator chooses t_A and μ that can affect the teacher's choices. In the formulation of Lemma 2, it is assumed that t_A and μ are exogenous, but in this model, when we consider the administrator's choice, these parameters become endogenous. Consequently, Lemma 2 is more accurately stated as Lemma 2A below.

Lemma 2A. *With endogeneity of t_A and μ , for all student profiles, there are two possible solutions to the teacher's optimal choice of instruction time t_{R1} : the lower bound t_A , and the upper bound $t_A + \mu$.*

The difference between Lemma 2 and Lemma 2A is that the teacher no longer has an interior solution when we endogenise t_A and μ . The administrator faces the budget constraint $w_1 t_A + w_2 \mu + w_3 Z_A = B$ where $w_2 \neq 0$. If $t_{R1} < t_A + \mu$, the administrator is paying the teacher w_2 for some units of μ that is not used for teaching. Thus, the administrator is better off lowering μ until $t_{R1} = t_A + \mu$, thereby eliminating any inte-

rior solution.⁶ The task for the rest of this section is then to determine how t_A and μ are chosen in a one-administrator-one-teacher case.

On the outset, it would be useful to restate that the administrator solves the following problem: $\max_{t_A, \mu, Z_A} \ln A + \alpha_1 \ln t_{S1} + \alpha_2 \ln t_{P1} + \alpha_3 \ln t_{R1} + \beta_A \ln Z_A$ s.t. $w_1 t_A + w_2 \mu + w_3 Z_A = B$

Using a reasonable assumption that $w_1 < w_2$, if the administrator wants the teacher to operate at the lower bound, then $\mu = 0$, since it is not profitable to pay w_2 for μ that is not used. If the administrator wants the teacher to operate on the upper bound, $t_A = 0$, since t_A and μ are perfect substitutes, and μ is the cheaper resource for production at the upper bound.

To choose between the two alternatives, the administrator has to decide which alternative will leave him better off. To achieve this we will consider two idealised cases where the administrator has complete autonomy over lesson time before interacting these results with the teacher's choice. Using a similar optimisation method, where solution set (7) corresponds to the lower bound cases and solution set (8) to the upper bound cases, the solutions are:

$$\frac{w_1 \beta_A}{B - w_1 t_A} + \frac{\alpha_1 + \alpha_2}{T - t_A - \varepsilon} = \frac{\alpha_3}{t_A} \quad (7A)$$

$$\frac{w_1 \beta_A}{B - w_1 t_A} + \frac{\alpha_1 + \alpha_2}{T - t_A} = \frac{\alpha_3}{t_A} \quad (7B)$$

$$\frac{w_1 \beta_A}{B - w_1 t_A} + \frac{\alpha_2}{T - t_A - 2\varepsilon} = \frac{\alpha_3}{t_A} \quad (7C)$$

$$\frac{w_2 \beta_A}{B - w_2 \mu} + \frac{\alpha_1 + \alpha_2}{T - \mu - \varepsilon} = \frac{\alpha_3}{\mu} \quad (8A)$$

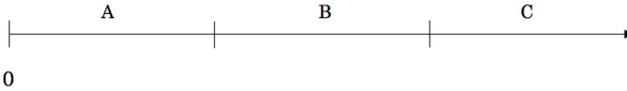
$$\frac{w_2 \beta_A}{B - w_2 \mu} + \frac{\alpha_1 + \alpha_2}{T - \mu} = \frac{\alpha_3}{\mu} \quad (8B)$$

⁶It should be noted that Lemma 2 is not completely irrelevant. (as it will be used again in Section 4.1) The teacher will still have three possible solutions if we extend beyond the one-administrator-one-teacher case, as the upper and lower bounds have to account for a heterogeneity of teachers, as is the result of contract theory.

$$\frac{w_2\beta_A}{B - w_2\mu} + \frac{\alpha_2}{T - \mu - 2\varepsilon} = \frac{\alpha_3}{\mu} \quad (8C)$$

In each set of solutions, (A), (B), and (C) correspond to the strong, average and weak students respectively. We denote the solutions to (6) as t_{R1}^* , to (7) as t_A^* , and to (8) as μ^* .⁷ The solutions to (7) and (8) show the administrator's optimal choice if he could exogenously determine learning Y subject to his budget constraint using either production factor t_A or μ . We know that $t_A^* < \mu^*$ since $w_1 > w_2$.

A simple number line would be helpful to observe how these solutions interact with t_{R1}^* .



With t_A^* and μ^* determined, there are three distinct regions where t_{R1}^* could go. Region A is where $t_{R1}^* \in (0, t_A^*)$. Region B is where $t_{R1}^* \in [t_A^*, \mu^*]$. Region C is where $t_{R1}^* > \mu^*$. It can be shown that the allocated t_{R1} will always be in region B, and the proof of Lemma 3 is in the Appendix.

Lemma 3. *The allocated t_{R1} will always be in region B where $t_{R1}^* \in [t_A^*, \mu^*]$.*

In region A, the administrator is better off paying w_1 to increase t_A until t_A^* is achieved, so the teacher's optimal solution is caught by the lower bound t_A^* . In region B, it is too costly for the administrator to use t_A , so he would pay the teacher w_2 for $\mu = t_{R1}^*$. In region C, even if the teacher wishes to teach further, the administrator would find it too costly to pay w_2 , and it would limit t_{R1} to μ^* , so the teacher's solution is caught by the upper bound μ^* . In summary, the administrator will use the lower bound for $t_{R1}^* \in A$ and the upper bound for $t_{R1}^* \in B, C$. The teacher has autonomy in lesson time

⁷The solutions show that we do not need to be concerned about the upper and lower limits of $t_{R1} \in (0, T - 2\varepsilon)$, since these limits appear in the denominator owing to the logarithmic formulation.

when $t_{R1}^* \in B$, and the administrator has complete autonomy otherwise.

F. Increasing Lesson Time

In summary, the sequential game-theoretic model has been solved by backward induction:

1. The student's optimal choice reflects whether he is a weak, average, or strong student.
2. For each student profile, the parent has a unique allocation.
3. Regardless of student profile, the teacher can have a choice at the lower bound, upper bound, or at an interior solution of her possible time allocations.
4. In this game with perfect information, and when the bounds are endogenously determined by the administrator, the teacher will no longer have an interior solution. The administrator chooses an upper and lower bound for the teacher.

The final section of this chapter will look at how the model explains the effectiveness of allocating more lesson time. There are two ways to interpret "allocating more lesson time". The first way is to think of the administrator increasing either the lower bound t_A or the upper bound μ , and the second way is to think of exogenously increasing lesson time t_{R1} . This model shows how, under both interpretations, the returns to more lesson time can be small.

Proposition 2. *The responsiveness of teaching time to the administrator's choices may be limited because:*

- *A marginal increase in the lower bound has no effect on optimal solutions operating on the upper bound.*

- A marginal increase in the upper bound has no effect on optimal solutions operating on the lower bound and solutions on the upper bound where $t_{R1} = t_{R1}^*$.

To show the effect of the administrator's choices, we will first consider variation in t_A in each region (with reference to the number line in Section 3.5), then the variation of μ in each region. Considering t_A first: in region A, an increase in t_A will cause t_{R1} to respond fully, and a decrease in t_A will cause t_{R1} to respond fully unless t_A falls sufficiently such that $t_A < t_{R1}^*$. In Regions B and C, t_A is already 0. A marginal increase will have no effect on t_{R1} , but if it is increased sufficiently, the existing solution can be caught by the lower bound t_A . This corresponds to Proposition 2a.

Consider μ : in region A, an increase in μ will have no effect, since the optimal solution is determined by t_A . In region B, an increase in μ will also have no effect, since the teacher already has autonomy over how much she teaches (i.e., $t_{R1} = t_{R1}^*$) and giving her more flexibility does not oblige her to increase her teaching hours. Notably, when t_{R1}^* is in region B, it is unaffected by t_A and μ . In region C, a marginal increase in μ will cause t_{R1} to respond fully, since the teacher desires to teach more. However, when μ increases sufficiently such that $\mu > t_{R1}^*$, t_{R1} will cease to respond. This corresponds to Proposition 2b. If t_{R1} does not respond to changes in t_A or μ , then such exogenous changes by the administrator would have no effect on Y .

Proposition 3. *The allocation of additional lesson time will have a small effect on learning if the productivities of the parent and of the student are high relative to the teacher's productivity i.e., α_1 and α_2 are large relative to α_3 .*

The intuition behind Proposition 3 largely arises from how the game is played. When the teacher exogenously increases her lesson time, there will be less time for the parent and student to allocate. Since the allocation is done by proportion (as in Lemma 1), there will be less learning time allocated by

the parent and the student i.e., t_{P1} and t_{S1} will fall. The role of these time allocations on learning is given by $t_{S1}^{\alpha_1} t_{P1}^{\alpha_2}$, and the fall in $t_{S1}^{\alpha_1} t_{P1}^{\alpha_2}$ will be larger for larger α_1 and α_2 . Thus, even when the increase in t_{R1} causes an overall increase in learning, the effect can be small when $t_{S1}^{\alpha_1} t_{P1}^{\alpha_2}$ falls drastically.

To show that Proposition 3 is true, we will just show the result for a strong student where a teacher operates on the lower bound, and similar arguments can be made analogously for the other cases. The logarithm of learning can be written as such:

$$\ln Y = \ln A + \alpha_1 \ln t_{S1} + \alpha_2 \ln t_{P1} + \alpha_3 \ln t_{R1}$$

For a strong student with a teacher choosing the lower bound,

$$\begin{aligned} \ln Y = \ln A + \alpha_1 \ln \left(\frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} \right) (T - t_A - \varepsilon) \\ + \alpha_2 \ln \left(\frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} \right) (T - t_A - \varepsilon) + \alpha_3 \ln t_A \end{aligned}$$

$$\frac{\delta \ln Y}{\delta t_A} = -\frac{\alpha_1 + \alpha_2}{T - t_A - \varepsilon} + \frac{\alpha_3}{t_A} \quad (9)$$

From (9), it is immediately apparent that $\frac{\delta \ln Y}{\delta t_A}$ increases with α_3 , and decreases with α_1 and α_2 .⁸ It is also notable that while $\frac{\delta \ln Y}{\delta t_A}$ can be small, it cannot be negative.⁹ Using (9) and (7A), we obtain $\frac{\delta \ln Y}{\delta t_A} = -\frac{\alpha_1 + \alpha_2}{T - t_A - \varepsilon} + \frac{\alpha_3}{t_A} = \frac{w_2 \beta_A}{Z_A w_3} > 0$. The intuition is that if investing in additional lesson time leads to negative returns on learning, the administrator would be better off investing the funds in her other objectives, such that utility can increase both from higher levels of the alternative objective and from better learning outcomes. Thus, the original allocation of funds would be such that t_A is below the

⁸ $\frac{\delta \ln Y}{\delta t_A}$ is the percentage increase in test score for a small increase in allocated time.

⁹ There would be negative returns only if the administrator does not optimise his objective function so the t_A allocated is already beyond the maximum point for learning.

optimal learning level. With comparative statics on α_1 and α_2 , Proposition 3 is shown.

It is notable that for a weak student at the lower bound, we will obtain the following differential:

$$\frac{\delta \ln Y}{\delta t_A} = -\frac{\alpha_2}{T - t_A - 2\varepsilon} + \frac{\alpha_3}{t_A}$$

Thus, although α_1 becomes irrelevant as the student already chooses the minimum time possible and teacher's additional time does not affect t_{S1} , the additional lesson time can still decrease the parent's input, which can be sizeable in α_2 .

IV. Comparative Statics

A. Why Does an Increase in Lesson Time Have a Small Effect on Learning?

From equation (9) and the existing propositions, there are at least three reasons that an increase in lesson time has a small effect on learning:

1. The α_3 parameter is small.
2. The teacher is not responsive to an increase in allocated time.
3. The productivities of the parent and of the student are high relative to the teacher's productivity.

The first reason is well-recognised in literature, and is consistent with the game-theoretic model. The second and third reasons are especially apparent in this proposed model, corresponding to Proposition 2 and Proposition 3 respectively.

Small α_3 Parameter

Proposition 3 has already highlighted the importance of α_3 in determining the effectiveness of additional classroom time with the teacher. While much emphasis is placed on the effectiveness of the teacher, α_3 essentially shows the productivity

of the teacher in helping the student learn, and this is a two-way process. A competent teacher's pedagogy might be better suited to strong students rather than weak students, so α_3 is still low for the weak student – this corresponds to Allensworth and Nomi's (2009) empirical findings. Consequently, α_3 more accurately captures the efficiency of the teacher-student interaction in producing results.

This result is consistent with existing literature on how teacher productivity is correlated with test scores. Cattaneo, Oggenfuss and Wolter (2016) show that students benefit to different extents from additional instruction time due to their differing aptitudes, which can be a determinant of α_3 . Kindron and Lindsay (2014) also highlight that programs that increased learning time improved academic outcomes when led by certified teachers, and this teacher competence can be captured by α_3 . When α_3 is small, we should expect the effect of more lesson time to be correspondingly small.

Teacher Responsiveness

When applying the model to more realistic settings, it should be noted that we would no longer have the idealised cases of $t_A = 0$ or $\mu = 0$, but we would expect t_A and μ to account for heterogeneity in teacher choices. Then, it would be possible that some teachers have an interior solution. Analogously, teachers with lower bound solutions would operate in region A; teachers with interior solutions in region B; and teachers with upper bound solutions in region C, and Proposition 2 would hold.

Proposition 2 highlights how changing the upper and lower bounds of teaching time would only affect actual lesson time under specific circumstances, and this would diminish the effect of reforms concerning teaching time. With heterogeneity of teachers and students in school, it is unlikely that a school-wide teaching time reform would affect all teachers. For instance, if a teacher already operates on the upper bound i.e., she is teaching some of her students beyond the minimum allocated time through supplementary classes, a school-wide in-

crease in t_A would be inconsequential for this group that she is already teaching. Thus, even if the teacher is productive in helping students learn, $\frac{\delta \ln Y}{\delta t_A}$ will be small. This result is significant for studies on school-wide reforms of instruction time. The studies cited typically use the school's official records of lesson time (Gandara, 2000), or students' estimate of their lesson time (such as in the PISA survey). However, these two measures tend to capture t_A rather than t_{R1} as students and official records count the periods allocated per week and the duration of each period. In a game-theoretic setting, we allow the teacher to choose an instruction time beyond t_A , and since $t_{R1} \geq t_A$, these studies tend to underestimate the actual effect of increased lesson time, especially for teachers operating on the lower bound when the aggregates are taken.

It may be argued that teachers conducting extra lessons (beyond what is allocated) are anomalies, but Proposition 2 will still be relevant when we look at a larger scale. At the school level, we assume that the teacher only teaches the amount of time that is allocated by the school. Then, t_A and $t_A + \mu$ would be the lower and upper bounds mandated by the district authorities or education ministry. The school would then choose t_{R1} for the teacher to teach. Under these assumptions, studies that measure school level teaching hours would have a more accurate representation of the effect of additional hours taught. This is in education reforms that aim to increase t_A for several schools. The argument from responsiveness is important when considering broad education policies such as setting term times without other education reforms.

There are some limitations of the responsiveness argument. Firstly, when the empirics account for actual lesson time t_{R1} , this account from responsiveness does not explain the low returns from education, since $t_A = t_{R1}$ at all values. Secondly, the responsiveness argument will be irrelevant if and only if the district authorities set lesson times that schools follow perfectly, and teachers follow the schools' lesson time perfectly (i.e., $\mu = 0 \forall t_{R1}^*$). This would not give room for any variation, which makes actual lesson time highly responsive to policy

changes.

Parent and Student Productivity

Proposition 3 states how the returns to additional lesson time can be low when the parent and student are very productive in learning. Since t_{P1} and t_{S1} will decrease when t_{R1} increases, the teacher can “crowd out” learning time allocated by the parent and the student. While it need not lead to negative returns in learning, it does result in a small increase in learning.

This result is empirically significant as regressions of most empirical studies on the effectiveness of additional lesson time attempt to control for family and student factors. However, an integral part of increasing lesson time is the parents’ and students’ response to this increase in lesson time through their own time allocation for learning. Gijsselaers et al (1995) found an inverse relationship between time for self-study and instructional time in their study on medical students, which is consistent with the argument on the scarcity of time.

B. How Can Learning Be Improved?

The determinants of learning are given in equation (2), and it would be helpful to restate them here to analyse how it can be increased. It is clear that Y varies positively with the exogenous A , the α parameters, and the time allocations as determined by the β parameters. Each of these components will be discussed.

$$Y = At_{S1}^{\alpha_1} t_{P1}^{\alpha_2} t_{R1}^{\alpha_3}$$

Exogenous A

As mentioned in Section 3.2, it is possible that A is a function of a student’s learning capacity, effort, and resources i.e., $A = A(C, e, S)$. Learning capacity and student effort are part of the exogenous factor, since a student with high aptitude and effort would transform time input into learning outputs efficiently regardless of where that time is spent. Resources

is more debatable, since good learning resources at home increases α_1 and α_2 , while good learning resources in school increases α_3 . Thus, the learning resources in A has to take a specific form such that it amplifies time input regardless of where that time is spent. Some examples might include stationery and accessible reference materials that the student carries with her both in school and at home.

How can educators increase A ? Learning capacity is what Anderson (2000) refers to as “aptitude”, which is consistent with the Carroll model, and this is difficult for the policy-maker to manipulate in the short run. However, encouraging the student to put in effort regardless of context can be taught and inculcated. Besides pedagogical initiatives, Levin and Tsang (1987) also note that effort can be induced through increasing the student’s pecuniary and non-pecuniary payoffs to Y . This could mean making learning more fun (non-pecuniary), or increasing the returns to education (pecuniary). In our model, increasing relative payoffs to Y in the student’s utility function is the same as decreasing β_S . In later discussion, the fall in β_S will be seen to increase t_{S1} because effort is treated exogenously, but if we allow effort to vary, the fall in β_S can also incentivise effort.

α Parameters

For the student, α_1 can be increased by teaching the student personal study skills such that she is able to increase her marginal productivity. α_1 can also be increased by providing the student with homework that enhances learning: Eren and Henderson (2011) found that this effectiveness can vary by subject. For the parent, α_2 is unlikely to change in the short run, since it is a measure of family productivity. The parent obtaining a more competent personal tutor might increase α_2 , but to the extent that t_{P1} is the parent’s personal tutoring of the student, we might observe a higher α_2 when the child becomes more educated and becomes a parent.

Increasing α_3 is probably the most commonly discussed in existing literature. Gromada and Shewbridge (2016) argue

that it is not the “allocated instruction time” that matters, but the actual learning time that counts. Out of the actual lesson time, they noted that the student is engaged only a fraction of that time, and out of which they are actually learning only for a fraction of it.¹⁰ Converting more of the allocated time (or lesson time) into actual learning time in their model is essentially increasing α_3 . Among others, Cotton (1989) suggested some ways that teachers and administrators can use allocated instructional time more effectively. For instance, teachers can reduce time spent on non-instructional activities and can choose tasks that help students experience success.

Time Allocations

If we look at the time solutions for an average student (and an interior solution for the teacher), it is evident that they are functions of the β parameters. In fact, the time spent t_{i1} varies negatively with β_i i.e., $\frac{\delta t_{i1}}{\delta \beta_i} < 0$. Thus, to increase the time input to increase learning Y , β_i has to decrease.

$$t_{S1} = \frac{\alpha_1}{\alpha_1 + \beta_S} T_S, \quad t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} T_P$$

The Cobb-Douglas function for utility in equation (1) effectively normalises Y , and treats β_i as how much more (or less) that agent values Z relative to Y . Thus, to lower β_i , the agent has to believe that other objectives are less valuable, or that maximising the student’s learning is more valuable. For the student, β_S is decreased by increasing the pecuniary and non-pecuniary benefits of learning. A parent might also reduce the student’s access to non-academic activities or convince the student that these activities are less valuable so that the student has less utility from other activities. For the parent, β_P

¹⁰Their time loss model can be accommodated into the game-theoretic model. Gromada and Shewbridge suggest that actual lesson time is a fraction of the allocated instruction time due to exogenous shocks such as snowstorms that can make a school close, thereby suggesting that $t_A > t_{R1}$. However, a proper incorporation of their point into the model is that snowstorms decrease both t_{R1} and t_A , since the minimum time expected of the teacher has fallen. Thus, for some initial $t_{R1} > t_A$, after a shock of ϵ , we have $t_{R1} - \epsilon > t_A - \epsilon$.

is decreased similarly by increasing the benefits of learning, and signalling these benefits to the parent. Since the parent typically wants the child to succeed, perceiving that academic activities are more valuable for the child and non-academic activities less valuable will decrease β_P . A similar argument may be made for a teacher. However, it is notable that if β_R is too high for the teacher, the administrator can simply increase t_A such that $t_{R1} = t_A$, but this means that the β_A parameter would matter. The interplay would then be similar to that discussed in Section 3.5.

V. Alternative Formulations

Thus far, under some assumptions in a sequential game, it has been shown that low returns of additional lesson time can be explained by a teacher's low productivity and responsiveness, and the relatively high productivity of a student's out-of-school learning time, and this thesis encapsulates Propositions 2 and 3. This chapter will drop some of the assumptions used in the setup and use alternative formulations of the model and show that the thesis is still robust to these alternative formulations.

The rest of this chapter will consider each of the following three assumptions in turn, and construct alternative formulations:

1. The parent is involved in the child's learning.
2. The α parameters are exogenous.
3. There is perfect information.

The question we want to ask in this section is: if we drop one of these assumptions, can low returns to additional lesson time still be explained by productivity and responsiveness?

A. Omitting the Parent

A large part of the model is reliant on the parent's input, and PISA data does show a sizeable out-of-school learning time.

However, one might still argue that the parent may not control the children's lives strictly, or that the parent's input is negligible.

If we omit the parent, then the learning function will take the following form:

$$\ln Y = \ln A + \alpha_1 \ln t_{S1} + \alpha_3 \ln t_{R1}$$

The student time constraint will then be: $t_{S1} + t_{S2} = T_S$, and $T_S + t_{R1} = T$. Using the results for strong, average and weak students respectively, we obtain the following differentials:

$$\frac{\delta \ln Y}{\delta t_{R1}} = -\frac{\alpha_1}{T - t_{R1} - \varepsilon} + \frac{\alpha_3}{t_{R1}} \quad (\text{Strong})$$

$$\frac{\delta \ln Y}{\delta t_{R1}} = -\frac{\alpha_1}{T - t_{R1}} + \frac{\alpha_3}{t_{R1}} \quad (\text{Average})$$

$$\frac{\delta \ln Y}{\delta t_{R1}} = \frac{\alpha_3}{t_{R1}} \quad (\text{Weak})$$

Evidently, when α_2 is eliminated, Proposition 3 will still hold for the strong and the average student, as high student productivity α_1 can lead to low returns to additional lesson time. However, Proposition 3 becomes irrelevant for the weak student, since higher t_{R1} cannot decrease t_{S1} any further. This means that low returns can still be explained by low α_3 , but not by higher α_1 for the weak student. Thus, the productivity explanation is still robust to this formulation, but it has to be qualified.

Other than the elimination of α_2 , the interaction between the administrator and the teacher is no different, so the responsiveness explanation would still be valid. Interestingly, Proposition 1 can now be applicable to the teacher, since a highly committed teacher may crowd out a strong student such that she becomes weak. Crowding out should not be viewed negatively because this implies high teacher productivity α_3 , which leads to better learning outcomes.

B. Endogenous Productivity

The basic model has assumed that the productivity parameters α are exogenous. However, it is possible to incorporate interaction effects in the model in the sense that more time spent with the teacher can allow a student to be more productive in his personal study time i.e., $\alpha_1 = \alpha_1(t_{R1})$. This is empirically possible through an inductive effect from the teacher.

In this simplified model, we can suppose a linear relationship between α_1 and t_{R1} , where $\alpha_1 = \gamma_1 t_{R1}$, where γ_1 can be interpreted as the student's exogenous productivity. Considering only the average student (the other two cases can be verified analogously), we have the following expression:

$$\begin{aligned} \ln Y &= \ln A + (\gamma_1 t_{R1}) \ln \left[\left(\frac{\alpha_1}{\alpha_1 + \beta_S} \right) \left(\frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} \right) (T - t_{R1}) \right] \\ &\quad + \alpha_2 \ln \left[\left(\frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} \right) (T - t_{R1}) \right] + \alpha_3 \ln t_{R1} \\ \ln Y &= C + \gamma_1 t_{R1} \ln (T - t_{R1}) + \alpha_2 \ln (T - t_{R1}) + \alpha_3 \ln t_{R1} \end{aligned}$$

Taking the first-order condition, we obtain:

$$\frac{\delta \ln Y}{\delta t_{R1}} = \gamma_1 \ln (T - t_{R1}) - \frac{\gamma_1 t_{R1} + \alpha_2}{T - t_{R1}} + \frac{\alpha_3}{t_{R1}}$$

Evidently, Proposition 3 is still robust as returns to lesson time varies positively with α_3 and varies negatively with α_2 . The role of the exogenous productivity of the student is more interesting. Differentiating the expression with respect to γ_1 , we obtain:

$$\frac{\delta}{\delta \gamma_1} \frac{\delta \ln Y}{\delta t_{R1}} = \ln (T - t_{R1}) - \frac{t_{R1}}{T - t_{R1}}$$

This implies that the effect of the exogenous productivity is still ambiguous, and depends on the value of t_{R1} .

$$\frac{\delta}{\delta t_{R1}} \frac{\delta}{\delta \gamma_1} \frac{\delta \ln Y}{\delta t_{R1}} = -\frac{1}{T - t_{R1}} - \frac{T}{(T - t_{R1})^2} < 0$$

Thus, when t_{R1} is sufficiently high, $\frac{\delta}{\delta\gamma_1} \frac{\delta \ln Y}{\delta t_{R1}} < 0$, and vice versa for a sufficiently low t_{R1} .¹¹ This implies that a highly productive student (with high γ_1) can still have low returns to additional lesson time when the teacher takes additional time. The intuition is that if the student has high productivity $\gamma_1 t_{R1}$, it still cannot be translated into learning output if he does not actually spend time studying, and higher t_{R1} causes t_{S1} to fall. Nonetheless, Proposition 3 is significantly weakened when interaction effects are included, as the additional lesson time increases the learning not just from the teacher's productivity, but also from increasing the student's productivity.

As with the case in Section V, part A, the key aspect of Proposition 3 is still robust, though it has to be qualified in light of the new formulation. Proposition 2 is still robust as the formulation does not affect the teacher-administrator interaction.

C. Imperfect Information

The specific form of information imperfection expounded on here is the inability of the teacher to know the student profile when making her choices. This is a significant modification to the game: the parent will still know the student profile, but from the teacher's perspective, the student profile is determined by chance. In the one-teacher-one-student formulation, the student can be strong, average, or weak each with some probability, and the teacher's choice must account for this uncertainty. This can be applied more realistically to a classroom, where there is a proportion of students in each category, and the teacher's choice has to cater to this mix to optimise learning. Modelling heterogeneity in the classroom will be the primary reference for this section.

Assuming that the result from Proposition 1 is irrelevant (i.e. the parent does not crowd out the student), the student profile is a function of α_1 and β_S . Holding β_S constant, we

¹¹Noting that we still require $t_{R1} < T - 2\epsilon$ for the average student, it becomes rather unlikely that t_{R1} is high enough for $\frac{\delta}{\delta\gamma_1} \frac{\delta \ln Y}{\delta t_{R1}} < 0$.

allow α_{1H} , α_{1M} , and α_{1L} for high, medium, and low values of α_1 , corresponding to the strong, average and weak students respectively i.e. $\alpha_{1H} > \alpha_{1M} > \alpha_{1L}$. Let the probability of having a strong student be π_1 , and a weak student be π_2 , so the average student will be $1 - \pi_1 - \pi_2$.

Finally, we require a fairly strong assumption for the expected utility function to avoid an unwieldy expression. We will suppose that learning is expressed as $\ln Y$ rather than Y . Then, the expected learning function can be formulated as such:

$$\begin{aligned} E(\ln Y) &= C + \pi_1(\alpha_{1H} + \alpha_2) \ln(T - t_{R1} - \varepsilon) \\ &\quad + (1 - \pi_1 - \pi_2)(\alpha_{1M} + \alpha_2) \ln(T - t_{R1}) \\ &\quad + \pi_2\alpha_2 \ln(T - t_{R1} - 2\varepsilon) + \alpha_3 \ln t_{R1} \end{aligned}$$

Again, we consider the effect of additional lesson time with the teacher by taking the derivative with respect to t_{R1} ,

$$\begin{aligned} \frac{\delta E(\ln Y)}{\delta t_{R1}} &= -\frac{\pi_1(\alpha_{1H} + \alpha_2)}{T - t_{R1} - \varepsilon} - \frac{(1 - \pi_1 - \pi_2)(\alpha_{1M} + \alpha_2)}{T - t_{R1}} \\ &\quad - \frac{\pi_2\alpha_2}{T - t_{R1} - 2\varepsilon} + \frac{\alpha_3}{t_{R1}} \end{aligned}$$

The result is still consistent with the predictions of Proposition 3, as the effect still depends on the α parameters. However, it would be interesting to look at how the returns to instructional time would change with π_1 and π_2 .

$$\begin{aligned} \frac{\delta}{\delta \pi_1} \frac{\delta E(\ln Y)}{\delta t_{R1}} &= \frac{(-\alpha_{1H} + \alpha_{1M})(T - t_{R1}) - (\alpha_{1M} + \alpha_2)\varepsilon}{(T - t_{R1} - \varepsilon)(T - t_{R1})} < 0 \\ \frac{\delta}{\delta \pi_2} \frac{\delta E(\ln Y)}{\delta t_{R1}} &= \frac{\alpha_{1M}(T - t_{R1} - \varepsilon) - 2\varepsilon\alpha_2}{(T - t_{R1} - 2\varepsilon)(T - t_{R1})} \end{aligned}$$

Before the analysis, it should be noted that the first equation assumes that the proportion of weak students is constant, so changes in the proportion of strong students is matched by a change in the proportion of average students. Similarly, the second equation assumes that the proportion of strong

students is constant, so changes in the proportion of weak students is matched by a change in the proportion of average students.

From the first result, it is evident that if a larger proportion of a class are strong students, the return to additional time with the teacher is unambiguously lower. This is simply due to the property that $\alpha_{1H} > \alpha_{1M}$, so having more students with α_{1H} implies that students are on balance more productive, and consistent with Proposition 3, the marginal return to additional lesson time is lower. The effect from π_2 , however, is more ambiguous. Having more weak students would mean higher returns to lesson time when α_{1M} is sufficiently high, so the fall in the mean of α_1 is more significant. Interestingly, when α_2 is very high, having more weak students could mean lower returns to additional lesson time: this occurs because t_{P1} is higher for a weak student than an average student, so if the parent is very productive, additional lesson time can still lower t_{P1} such that returns are low. When the administrator has the same imperfect information as the teacher, the optimisation problems will be similar, and the results of Proposition 2 will be robust.

The crux of the arguments in Chapter V is that the basic mechanism can be incorporated into alternative formulations. Proposition 2 works because of how the teacher chooses time allocations relative to the administrator's choice. Proposition 3 works because time is a scarce resource and spending more time in the classroom necessarily means less time elsewhere, so the relative productivities of time spent becomes relevant. Alternative formulations might modify some aspects of these mechanisms, but the core thesis is robust as long as the structure of the sequential game remains.

VI. Concluding Remarks

This paper has shown that low returns of additional lesson time can be explained through a teacher's low productivity and responsiveness, and by decreasing a student's out-of-school learning time, when we formulate student learning time as

a sequential game-theoretic model. The conclusion is robust to some alternative formulations such as omitting the parent, endogenous productivity, and imperfect information. These formulations are non-exhaustive, and the model can be easily extended and tested against other variants.

The theoretical model can also be extended to include funding choice, teacher's complementary activities, and the complementarity of non-academic activities, for instance. In funding choice, the budget could be spent on more instructional hours t_{R1} , or on better facilities that will improve the teacher's productivity i.e. $\alpha_3 = \gamma_3 f$, where f is the level of facilities investment. A similar problem would be to assume that the teacher only wishes to maximise Y , and her time spent on non-teaching activities can make the teaching more productive. This means that she still has the constraint $t_{R1} + t_{R2} = T_R$, but $\alpha_3 = \gamma_3 t_{R2}$. Intuitively, this means that outside-classroom activities such as marking students' work, handling disciplinary cases etc. can help her become more productive in the classroom. The complementarity of non-academic activities could take the form of $\alpha_1 = \gamma_1 t_{S2}$, thereby working on the assumption that activities such as music and sport might sharpen the mind to make the student more productive. While these are interesting extensions of the model, we have already shown how the thesis is already robust alternative formulations, so they are not discussed in detail.

This paper's main contribution is in tying four active players in a student's learning into a coherent framework. It amalgamates well-studied teacher-student interfaces, student optimisation decisions, and parent-student interfaces. Rather than treating the student as a passive object of the teacher's and parent's optimisation problems, as is the assumption of many empirical studies, this model shows how a student's active preferences and decisions can affect his learning outcome. Similarly, the parent is no longer a passive observer of the teacher-student interface that produces learning – the parent also has an active role to play in a child's education. Through the activity of other players, it also shows how changes at a

policy level could have little effect due to low responsiveness when agents are not operating at the relevant boundary.

One limitation of this paper is the use of restrictive assumptions and a functional form. They were largely used in this paper for simplicity, since using general forms could make expressions in the four-player game rather unwieldy. While the Cobb-Douglas specification is generally used in theoretical papers, it has paradigmatic implications that are non-trivial. For one, it contradicts the original Carroll model. The Carroll model (1963) suggests that degree of learning = $f(\text{time actually spent} / \text{time needed})$, and learning is maximised when the argument approaches one. The student stops learning when one of three things happen: (1) attained a specified standard of competence (2) spends an amount of time determined by perseverance (3) ran out of time allocated. The first phenomenon that a student stops learning after she has “learned” the piece of information (i.e. attained a particular standard) implies that, after sufficient time has been spent, the returns to an additional unit of time in the classroom for a strong student is asymptotically zero while the returns to an additional unit of time for a weak student could still be high. However, in the Cobb-Douglas formulation, with α_1 being higher for the strong student than the weak student, the returns to additional time is still higher for the strong student than the weak student. More importantly, the Cobb-Douglas formulation has no limit to learning – it implies that learning Y can be infinite, which is at odds with the Carroll formulation. As such, further theoretical research can be done to observe whether the results are robust to general functional forms and with less restrictive assumptions.

Further research can also be done on the empirics of this model. It is possible to collect survey data from students on how their time is allocated. While there are already studies on students’ self-reported lesson time and self-study time among tertiary students (see Grave, 2011; Gijssels et al, 1995), the literature is less extensive among primary and secondary students. Under more restrictive assumptions, big data (such as

PISA) can also be used through fixed effects estimation similar to Lavy's (2015) approach. The empirics also can help to estimate or verify a more accurate functional form for learning.

References

- [1] Anderson, L. W. (2000). "Time, learning and school reform: a conceptual framework" in P. Gandara (ed.) *The dimensions of time and the challenges of school reform*. New York: State University of New York Press.
- [2] Bloom, B.S. (1968). "Learning for mastery". *Evaluation Comment* vol. 1(2) pp. 4-14.
- [3] Brown and Saks (1981). "The microeconomics of schooling", in Berliner, D.C. (ed.) *Review of research in education*. Washington, DC: American Educational Research Association.
- [4] Brown and Saks. (1984). "An economic view of the acquisition of reasoning skills: agenda for research in the information age", *Review of Educational Research* vol. 54(4) pp. 560-576.
- [5] Carroll, J.B. (1963). "A model for school learning", *Teachers College Record* vol. 64 pp. 723-733.
- [6] Cattaneo, Oggenfuss, Wolter. (2016). "The more the better? The Impact of Instructional Time on Student Performance," IZA Discussion Paper No. 9797.
- [7] Coates, D. (2003). "Educational production functions using instructional time as an input", *Education Economics* vol. 11(3) pp.273-92.
- [8] Cotton, K. (1989). "Educational time factors", *School Improvement Research Series (SIRS), Close-Up 8*, Northwest regional Educational Laboratory, Portland.

- [9] Eren, O. and Henderson D. (2011). “Are we wasting our children’s time by giving them more homework?” *Economics of Education Review*, vol. 30(5) pp. 950-961.
- [10] Gandara, P. (2000). *The dimensions of time and the challenges of school reform*. New York: State University of New York Press.
- [11] Gijsselaers, W.H. and Schmidt, H.G. (1995). “Effects of quantity of instruction on time spent on learning and achievement”, *Educational Research and Evaluation*, vol. 1(2), pp.183-201.
- [12] Grave, B. S. (2011) “The effect of student time allocation on academic achievement”, *Education Economics* vol. 19(3) pp. 291-310.
- [13] Gromada and Shewbridge. (2016). “Student learning time: a literature review.” OECD Education Working Papers No 127.
- [14] Kindron and Lindsay. (2014). “The effects of increased learning time on student academic and non-academic outcomes: Findings from a meta-analytic review” (REL 2014-015). Washington, DC: US Department of Education, Institute of Education Sciences.
- [15] Lavy, V. (2015). “Do differences in schools’ instruction time explain international achievement gaps? Evidence from developed and developing countries”, *The Economic Journal*, vol. 125 pp. F397–F424.
- [16] Lazear, F. P. (2001). “Educational production” *Quarterly Journal of Economics*, vol. 116(3), pp. 777-803.
- [17] Levin, H. M. and Tsang, M. C. (1987). “The Economics of Student Time”, *Economics of Education Review*, vol. 6(4) pp.357-364.
- [18] Pischke, J. S. (2007). “The impact of length of the school year on student performance and earnings: evidence from

- the German short school years,” *The Economic Journal*, vol. 117(4), pp.1216-1242.
- [19] Rivkin, S. G. and Schiman, J.C. (2015). “Instruction time, classroom quality, and academic achievement”, *The Economic Journal*, vol. 125, F425-F448.
- [20] Schalock, Cowart and Staebler. (1993). “Teacher productivity revisited: definition, theory, measurement, and application”, *Journal of Personnel Evaluation in Education* vol. 7 pp. 179-196.
- [21] Stoll, L., Fink, D., and Earl, L. (2003). *It’s about learning and it’s about time: What’s in it for schools?*. London: Routledge Falmer.
- [22] Todd, P. E. and Wolpin, K. I. (2003). “On the Specification and Estimation of the Production Function for Cognitive Achievement”, *The Economic Journal*, vol. 113(1) pp. F3–F33.
- [23] Woessmann, L. (2010). “Institutional determinants of school efficiency and equity: German states as a microcosm for OECD countries”, *Journal of Economics and Statistics*, vol. 230 pp. 234-270.

Appendix

Summary of Solutions

	$Y = A t_{S1}^{\alpha_1} t_{P1}^{\alpha_2} t_{R1}^{\alpha_3}$		
	Strong	Average	Weak
Student t_{S1}	$T_S - \varepsilon$	$\frac{\alpha_1}{\alpha_1 + \beta_S} T_S$	ε
Parent t_{P1}	$\frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$	$\frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} T_P$	$\frac{\alpha_2}{\alpha_2 + \beta_P} (T_P - 2\varepsilon)$
Teacher t_{R1}	$\frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} + \frac{\beta_R}{T_R - t_{R1}}$ $= \frac{\alpha_3}{t_{R1}}$	$\frac{\alpha_1 + \alpha_2}{T - t_{R1}} + \frac{\beta_R}{T_R - t_{R1}} = \frac{\alpha_3}{t_{R1}}$	$\frac{\alpha_2}{T - t_{R1} - 2\varepsilon}$ $+ \frac{\beta_R}{T_R - t_{R1}} = \frac{\alpha_3}{t_{R1}}$

Lemma 1. *The parent has a unique optimal solution to each of the three student profiles:*

For the strong student: $t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$, $t_{P2} = \frac{\beta_P}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$, and $T_S = \varepsilon + \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon)$

For the average student: $t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} T_P$, $t_{P2} = \frac{\beta_P}{\alpha_1 + \alpha_2 + \beta_P} T_P$, and $T_S = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} T_P$

For the weak student: $t_{P1} = \frac{\alpha_2}{\alpha_2 + \beta_P} (T_P - 2\varepsilon)$, $t_{P2} = \frac{\beta_P}{\alpha_2 + \beta_P} (T_P - 2\varepsilon)$, $T_S = 2\varepsilon$

Proof:

The parent's general problem is $\max_{t_{P1}, t_{P2}} \ln A + \alpha_1 \ln t_{S1} + \alpha_2 \ln t_{P1} + \alpha_3 \ln t_{R1} + \beta_P \ln t_{P2}$ s.t. $t_{P1} + t_{P2} + T_S = T_P$ and $T_S \geq 2\varepsilon$. We will then consider the three cases according to Definition 1. A strong student will choose $t_{S1} = T_S - \varepsilon$; an average student will choose $t_{S1} = \frac{\alpha_1}{\alpha_1 + \beta_S} T_S$; and a weak student will choose $t_{S1} = \varepsilon$.

Parent's Problem for weak student:

Substituting $t_{S1} = \varepsilon$ and using C to denote a catch-all variable for parameters that do not affect the parent's decision, we can rewrite the problem as:

$$\max_{t_{P1}, t_{P2}} C + \alpha_1 \ln \varepsilon + \alpha_2 \ln t_{P1} + \beta_P \ln t_{P2} \text{ s.t.}$$

$$t_{P1} + t_{P2} + T_S = T_P \text{ and } T_S \geq 2\varepsilon$$

$$L = C + \alpha_1 \ln \varepsilon + \alpha_2 \ln t_{P1} + \beta_P \ln t_{P2} - \lambda(t_{P1} + t_{P2} + T_S - T_P)$$

First-order conditions are given by:

$$L'_{t_{P1}} = \frac{\alpha_2}{t_{P1}} - \lambda = 0$$

$$L'_{t_{P2}} = \frac{\beta_P}{t_{P2}} - \lambda = 0$$

The variable T_S does not feature in the first-order conditions as the parent already knows that the student will choose the minimum t_{S1} possible. Consequently, to maximise the student's learning, the parent will minimise the time given to the student, so $T_S = 2\varepsilon$.

Solving the simultaneous equations, we obtain:

$$t_{P1} = \frac{\alpha_2}{\alpha_2 + \beta_P}(T_P - 2\varepsilon), t_{P2} = \frac{\beta_P}{\alpha_2 + \beta_P}(T_P - 2\varepsilon), T_S = 2\varepsilon$$

Parent's Problem for average student:

Substituting $t_{S1} = \frac{\alpha_1}{\alpha_1 + \beta_S}T_S$, we can rewrite the problem as:

$$\max_{t_{P1}, t_{P2}} C + \alpha_1 \ln T_S + \alpha_2 \ln t_{P1} + \beta_P \ln t_{P2} \text{ s.t.}$$

$$t_{P1} + t_{P2} + T_S = T_P \text{ and } T_S \geq 2\varepsilon$$

$$L = C + \alpha_1 \ln T_S + \alpha_2 \ln t_{P1} + \beta_P \ln t_{P2} - \lambda(t_{P1} + t_{P2} + T_S - T_P)$$

First-order conditions are given by:

$$L'_{t_{P1}} = \frac{\alpha_2}{t_{P1}} - \lambda = 0$$

$$L'_{t_{P2}} = \frac{\beta_P}{t_{P2}} - \lambda = 0$$

$$L'_{T_S} = \frac{\alpha_1}{T_S} - \lambda = 0$$

Solving the equations simultaneously, we obtain:

$$t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P}T_P, t_{P2} = \frac{\beta_P}{\alpha_1 + \alpha_2 + \beta_P}T_P, \text{ and } T_S = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P}T_P$$

However, this assumes that the constraint $T_S \geq 2\varepsilon$ does not bind. When α_2 and β_P are sufficiently large, it is possible that $\frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P}T_P < 2\varepsilon$, so the constraint will bind. A binding constraint implies that $T_S = 2\varepsilon$, so the solution will be the same as the weak student case of: $t_{P1} = \frac{\alpha_2}{\alpha_2 + \beta_P}(T_P - 2\varepsilon)$, $t_{P2} = \frac{\beta_P}{\alpha_2 + \beta_P}(T_P - 2\varepsilon)$, $T_S = 2\varepsilon$. But if we obtain $T_S = 2\varepsilon$, the student must choose $t_{S1} = \varepsilon$, and this corresponds to the definition of a "weak" student rather than an

“average” student. Thus, the “average” student still yields a unique solution.

Parent’s Problem for strong student:

Substituting $t_{S1} = T_S - \varepsilon$, we can rewrite the problem as:

$$\max_{t_{P1}, t_{P2}} C + \alpha_1 \ln(T_S - \varepsilon) + \alpha_2 \ln t_{P1} + \beta_P \ln t_{P2} \text{ s.t.}$$

$$t_{P1} + t_{P2} + T_S = T_P \text{ and } T_S \geq 2\varepsilon$$

$$L = C + \alpha_1 \ln(T_S - \varepsilon) + \alpha_2 \ln t_{P1} + \beta_P \ln t_{P2} - \lambda(t_{P1} + t_{P2} + T_S - T_P)$$

First-order conditions are given by:

$$L'_{t_{P1}} = \frac{\alpha_2}{t_{P1}} - \lambda = 0$$

$$L'_{t_{P2}} = \frac{\beta_P}{t_{P2}} - \lambda = 0$$

$$L'_{T_S} = \frac{\alpha_1}{T_S - \varepsilon} - \lambda = 0$$

Solving the equations simultaneously, we obtain:

$$t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P}(T_P - \varepsilon), t_{P2} = \frac{\beta_P}{\alpha_1 + \alpha_2 + \beta_P}(T_P - \varepsilon),$$

$$\text{and } T_S = \frac{\varepsilon + \alpha_1}{\alpha_1 + \alpha_2 + \beta_P}(T_P - \varepsilon)$$

However, this assumes that the constraint $T_S \geq 2\varepsilon$ does not bind. When α_2 and β_P are sufficiently large, it is possible that $\frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P}(T_P - \varepsilon) < \varepsilon$, so the constraint will bind. Using a similar argument as the average student, a binding constraint implies that $T_S = 2\varepsilon$. But if we obtain $T_S = 2\varepsilon$, the student must choose $t_{S1} = \varepsilon$, and this corresponds to the definition of a “weak” student rather than an “strong” student. Thus, the “strong” student still yields a unique solution as shown in the proposition.

With the consideration of these three cases, the uniqueness implied by Lemma 1 follows.

Lemma 2. *For all student profiles, there are three possible solutions to the teacher’s optimal choice of instruction time t_{R1} : an interior solution, the lower bound t_A , and the upper bound $t_A + \mu$.*

Proof:

The optimisation problem is:

$$\max_{t_{R1}, t_{R2}} \ln A + \alpha_1 \ln t_{S1} + \alpha_2 \ln t_{P1} + \alpha_3 \ln t_{R1} + \beta_R \ln t_{R2} \quad \text{s.t.}$$

$$t_A \leq t_{R1} \leq t_A + \mu \quad \text{and} \quad t_{R1} + t_{R2} = T_R$$

Teacher's Problem for strong student:

$$\text{Substituting } t_{S1} = T_S - \varepsilon = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon) \text{ and } t_{P1} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta_P} (T_P - \varepsilon),$$

$$\max_{t_{R1}, t_{R2}} C + \alpha_1 \ln (T_P - \varepsilon) + \alpha_2 \ln (T_P - \varepsilon) + \alpha_3 \ln t_{R1} + \beta_R \ln t_{R2}$$

s.t. $t_A \leq t_{R1} \leq t_A + \mu$ and $t_{R1} + t_{R2} = T_R$. Further substituting $T_P = T - t_{R1}$,

$$L = C + \alpha_1 \ln (T - t_{R1} - \varepsilon) + \alpha_2 \ln (T - t_{R1} - \varepsilon) + \alpha_3 \ln t_{R1} + \beta_R \ln t_{R2}$$

$$- \lambda (t_{R1} + t_{R2} - T_R)$$

First-order conditions:

$$L'_{t_{R1}} = -\frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} + \frac{\alpha_3}{t_{R1}} - \lambda = 0$$

$$L'_{t_{R2}} = \frac{\beta_R}{t_{R2}} - \lambda = 0$$

Solving them simultaneously,

$$\frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} + \frac{\beta_R}{T_R - t_{R1}} = \frac{\alpha_3}{t_{R1}}$$

Teacher's Problem for average student:

$$\max_{t_{R1}, t_{R2}} C + \alpha_1 \ln (T - t_{R1}) + \alpha_2 \ln (T - t_{R1}) + \alpha_3 \ln t_{R1} + \beta_R \ln t_{R2} \quad \text{s.t.}$$

$$t_A \leq t_{R1} \leq t_A + \mu \text{ and } t_{R1} + t_{R2} = T_R$$

$$L = C + \alpha_1 \ln T - t_{R1} + \alpha_2 \ln T - t_{R1} + \alpha_3 \ln t_{R1} + \beta_R \ln t_{R2} - \lambda (t_{R1} + t_{R2} - T_R)$$

First-order conditions:

$$L'_{t_{R1}} = -\frac{\alpha_1 + \alpha_2}{T - t_{R1}} + \frac{\alpha_3}{t_{R1}} - \lambda = 0$$

$$L'_{t_{R2}} = \frac{\beta_R}{t_{R2}} - \lambda = 0$$

Solving them simultaneously,

$$\frac{\alpha_1 + \alpha_2}{T - t_{R1}} + \frac{\beta_R}{T_R - t_{R1}} = \frac{\alpha_3}{t_{R1}}$$

Teacher's Problem for weak student:

$$\max_{t_{R1}, t_{R2}} C + \alpha_1 \ln(T - t_{R1} - 2\varepsilon) + \alpha_2 \ln(T - t_{R1} - 2\varepsilon) + \alpha_3 \ln t_{R1} \\ + \beta_R \ln t_{R2} \text{ s.t. } t_A \leq t_{R1} \leq t_A + \mu \text{ and } t_{R1} + t_{R2} = T_R$$

$$L = C + \alpha_1 \ln T - t_{R1} - 2\varepsilon + \alpha_2 \ln T - t_{R1} - 2\varepsilon + \alpha_3 \ln t_{R1} \\ + \beta_R \ln t_{R2} - \lambda(t_{R1} + t_{R2} - T_R)$$

First-order conditions:

$$L'_{t_{R1}} = -\frac{\alpha_1 + \alpha_2}{T - t_{R1} - 2\varepsilon} + \frac{\alpha_3}{t_{R1}} - \lambda = 0$$

$$L'_{t_{R2}} = \frac{\beta_R}{t_{R2}} - \lambda = 0$$

Solving them simultaneously,

$$\frac{\alpha_2}{T - t_{R1} - 2\varepsilon} + \frac{\beta_R}{T_R - t_{R1}} = \frac{\alpha_3}{t_{R1}}$$

Comparative Statics using strong student:

$$\alpha_1 = (T - t_{R1} - \varepsilon) \left(\frac{\alpha_3}{t_{R1}} - \frac{\beta_R}{T_R - t_{R1}} - \frac{\alpha_2}{T - t_{R1} - \varepsilon} \right)$$

$$\frac{\delta \alpha_1}{\delta t_{R1}} = (-1) \left(\frac{\alpha_3}{t_{R1}} - \frac{\beta_R}{T_R - t_{R1}} - \frac{\alpha_2}{T - t_{R1} - \varepsilon} \right) \\ + (T - t_{R1} - \varepsilon) \left(-\frac{\alpha_3}{t_{R1}^2} - \frac{\beta_p}{(T_R - t_{R1})^2} - \frac{\alpha_2}{(T - t_{R1} - \varepsilon)^2} \right) < 0$$

$$\alpha_2 = (T - t_{R1} - \varepsilon) \left(\frac{\alpha_3}{t_{R1}} - \frac{\beta_R}{T_R - t_{R1}} - \frac{\alpha_1}{T - t_{R1} - \varepsilon} \right)$$

$$\begin{aligned} \frac{\delta\alpha_2}{\delta t_{R1}} &= (-1) \left(\frac{\alpha_3}{t_{R1}} - \frac{\beta_R}{T_R - t_{R1}} - \frac{\alpha_1}{T - t_{R1} - \varepsilon} \right) \\ &+ (T - t_{R1} - \varepsilon) \left(-\frac{\alpha_3}{t_{R1}^2} - \frac{\beta_p}{(T_R - t_{R1})^2} - \frac{\alpha_1}{(T - t_{R1} - \varepsilon)^2} \right) < 0 \\ \alpha_3 &= t_{R1} \left(\frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} + \frac{\beta_R}{T_R - t_{R1}} \right) \end{aligned}$$

$$\begin{aligned} \frac{\delta\alpha_3}{\delta t_{R1}} &= \left(\frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} + \frac{\beta_R}{T_R - t_{R1}} \right) \\ &+ t_{R1} \left(\frac{\alpha_1 + \alpha_2}{(T - t_{R1} - \varepsilon)^2} + \frac{\beta_R}{(T_R - t_{R1})^2} \right) > 0 \\ \beta_R &= (T_R - t_{R1}) \left(\frac{\alpha_3}{t_{R1}} - \frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} \right) \end{aligned}$$

$$\begin{aligned} \frac{\delta\beta_R}{\delta t_{R1}} &= (-1) \left(\frac{\alpha_3}{t_{R1}} - \frac{\alpha_1 + \alpha_2}{T - t_{R1} - \varepsilon} \right) \\ &+ (T_R - t_{R1}) \left(-\frac{\alpha_3}{t_{R1}^2} - \frac{\alpha_1 + \alpha_2}{(T - t_{R1} - \varepsilon)^2} \right) < 0 \end{aligned}$$

Noting that $\frac{dy}{dx} > 0$ iff $\frac{dx}{dy} > 0$, and $\frac{dy}{dx} < 0$ iff $\frac{dx}{dy} < 0$, we have:

$$\frac{\delta t_{R1}}{\delta \alpha_1}, \frac{\delta t_{R1}}{\delta \alpha_2}, \frac{\delta t_{R1}}{\delta \beta_R} < 0 \quad \text{and} \quad \frac{\delta t_{R1}}{\delta \alpha_3} > 0$$

For a sufficiently high α_1, α_2 and β_R and sufficiently low α_3 , there will be a lower bound solution at t_A . For a sufficiently low α_1, α_2 and β_R and sufficiently high α_3 , there will be an upper bound solution at $t_A + \mu$. By analogy, this relation holds in all three student profiles, and we have proven Lemma 2.

Lemma 3. *The allocated t_{R1} will always be in region B where $t_{R1} \in [t_A^*, \mu^*]$.*

Proof:

To prove Lemma 3, we will use the case of an average student, and argument by analogy can be made for the other two cases.



Proving the upper bound is straightforward. If $t_{R1}^* > \mu^*$, the administrator is unwilling to pay for such a high t_{R1}^* even if using the cheapest resource, since its utility function is quasi-concave and maximum is achieved at μ^* . Thus, all solutions in C will move to μ^* .

In the one-teacher-one-administrator case, the administrator has to choose between $t_A = 0$ and $\mu = 0$. Its optimisation problem (where C collects terms that do not contain t_{R1} is:

$$\max C + (\alpha_1 + \alpha_2)(T - t_{R1}) + \alpha_3 \ln t_{R1} + \beta_A \ln Z_A \quad \text{s.t.} \quad w_1 t_A + w_2 \mu + w_3 Z_A = B$$

If the teacher has full autonomy ($t_A = 0$), then, denoting outcome as \tilde{U} ,

$$\ln \tilde{U} = C + (\alpha_1 + \alpha_2)(T - t_{R1}^*) + \alpha_3 \ln t_{R1}^* + \beta_A \ln \left(\frac{B - w_2 t_{R1}^*}{w_3} \right)$$

If the administrator has full autonomy ($\mu = 0$), then, denoting outcome as U^+ ,

$$\ln U^+ = C + (\alpha_1 + \alpha_2)(T - t_A) + \alpha_3 \ln t_A + \beta_A \ln (B - w_1 t_A w_3)$$

Let $D = \ln \tilde{U} - \ln U^+$. The administrator would be better off giving teacher full autonomy *iff* $D > 0$, and we are interested in finding out how D changes with t_{R1}^* .

Since t_{R1}^* does not feature in $\ln U^+$, $\frac{\delta D}{\delta t_{R1}^*} = \frac{\delta \ln \tilde{U}}{\delta t_{R1}^*}$. Thus,

$$\frac{\delta D}{\delta t_{R1}^*} = -\frac{w_2 \beta_A}{B - w_2 t_{R1}^*} - \frac{\alpha_1 + \alpha_2}{T - t_{R1}^*} + \frac{\alpha_3}{t_{R1}^*}$$

We know that D is concave because:

$$\frac{\delta}{\delta t_{R1}^*} \left(\frac{\delta D}{\delta t_{R1}^*} \right) = -\frac{w_2^2 \beta_A}{(B - w_2 t_{R1}^*)^2} - \frac{\alpha_1 + \alpha_2}{(T - t_{R1}^*)^2} - \frac{\alpha_3}{t_{R1}^{*2}} < 0$$

This implies that $\frac{\delta D}{\delta t_{R1}^*}$ continuously decreases as t_{R1}^* increases. Thus, $\frac{\delta D}{\delta t_{R1}^*} > 0$ for small values of t_{R1}^* and $\frac{\delta D}{\delta t_{R1}^*} < 0$ for large values of t_{R1}^* , and $\frac{\delta D}{\delta t_{R1}^*} = 0$ at a particular value of t_{R1}^* . We are able to find the particular value of t_{R1}^* such that $\frac{\delta D}{\delta t_{R1}^*} = 0$, by setting:

$$\frac{\delta D}{\delta t_{R1}^*} = -\frac{w_2 \beta_A}{B - w_2 t_{R1}^*} - \frac{\alpha_1 + \alpha_2}{T - t_{R1}^*} + \frac{\alpha_3}{t_{R1}^*} = 0$$

This equation is the same as (8B), so this point essentially occurs at μ^* , which is the upper bound of region B. Thus, for all optimal solutions in region A and B, $\frac{\delta D}{\delta t_{R1}^*} > 0$.

It is straightforward to see that $D = 0$ when $t_A^* = t_{R1}^*$. Since $\frac{\delta D}{\delta t_{R1}^*} > 0$, $D < 0$ in region A and $D > 0$ in region B. Thus, whenever the solution is in region A, the administrator is better off taking full autonomy and allocating t_A , and it has already been established that t_A optimises the administrator's utility at t_A^* , so the lowest possible allocation of t_{R1}^* will be at t_A^* .

When the solution is in region B, the administrator is better off with the teacher taking autonomy, and his paying the teacher accordingly, and the solution for t_{R1} will exist in region B.

Thus, Lemma 3 is shown.

Power, Expected Value, and Excessive Risk-Taking

Johnny Fulfer

Advisor: Dr. Peter Maille

Abstract

To investigate the influence of the sense of psychological power on decision making under risk, we conducted an experiment with 60 participants and found evidence which suggests that a heightened sense of power results in excessive risk-taking. We primed the experimental group with a high-power mindset and the control group with a neutral mindset and asked participants to answer a series of hypothetical questions on expected value. The results of a chi-square test suggest that participants who were primed with a heightened sense of power focus on high but unlikely rewards. When the potential reward was greater than the loss, power-primed people were swayed towards the rewards despite the unfavorable probabilities. We propose that there is a link between a heightened sense of power and suboptimal economic decision making, and model this in the Prospect Theory framework.

I. Introduction

Expected Utility Theory originally formulated by Daniel Bernoulli (1954) in the 18th century, suggests that individuals calculate risks with complete accuracy. Kahneman and Tversky (1979) critiqued this theory, arguing that people increase or decrease utility with respect to a reference point. Expected Utility Theory uses final states of wealth in order to compare utility, while Kahneman's and Tversky's (1979) Prospect Theory assesses the relative value of utility. Prospect Theory relies on two phases in the decision-making process. The first is an editing phase, which involves how individuals mentally organize and simplify information. The second is the evaluation phase, in which individuals choose the prospect which has the greatest value (Kahneman and Tversky, 1979). They fur-

ther argue that individuals tend to weigh prospects that seem more likely as a certainty rather than perceiving the prospect based on its actual probability. Beyond overweighting perceived probabilities, individuals also simplify prospects, isolating factors that are unique and discarding factors that are common to each prospect, a process Kahneman and Tversky (1979) call the isolation effect.

Thaler and Johnson (1990) developed the House Money Effect in conjunction with Prospect Theory, which suggests that prior gains and losses influence decision making under risk. Barberis, Haug, and Santos (2001) further support this by arguing that capital gains in the stock market increase risk taking, while losses lead to risk aversion. More recently, psychologists Anderson and Galinsky (2006) argue that individuals with a heightened sense of power tend to have more access to rewards and “encounter less interference” in the pursuit of those rewards (p. 513). They further argued that the possession of power may lead decision makers to “pay more attention to the potential payoffs inherent in risky actions and devote less attention to the potential dangers” (p. 512).

In this paper, we investigate the role of power in decision making under risk. Emerson (1962) and Thibaut and Kelly (1959) defined power as a social relation characterized by the ability to influence others. Those with a heightened sense of power are more likely to take action (see Galinsky et al., 2003; Keltner et al., 2003; Magee, Galinsky, and Gruefeld, 2007), and view the world “through an instrumental lens,” approaching those who are useful to them (Gruenfeld, Inesi, Magee, and Galinsky, 2008, p. 125). While Smith and Trope (2006) found that power creates a more global perspective, Inesi (2010) argues that power reduced the psychological impact of perceived losses. Rucker, Galinsky, and Dubois (2012) analyze power as a psychological state, which originates from cognitive, structural, and physical factors. More recently, Sturm and Antonakis (2015) conclude that the possession of micro-oriented power in its various forms (psychological, interpersonal, or structural) has the potential to change the individual

with power, although they recognize the impacts of power are not fully understood.

Anderson and Galinsky (2006) found that “the sense of power increases optimism in perceiving risks and leads to riskier behavior” (p. 511). They primed participants with a “high-power mind set” which led the subjects to be more optimistic about the outcome of risky behavior. The researchers asked participants to report their beliefs of how powerful they were in their relationships with others. Following this, participants were asked to estimate the chance they would experience 15 different life events, including, “Having your achievements displayed in the newspaper, having your home double in value in 10 years,” and 13 other similar questions (p. 516). They found that the sense of power that participants reported was related to how optimistic they were in their perception of future events. For example, participants that reported being more powerful in a social context were more optimistic about avoiding turbulence on an airplane, which is out of their control. Anderson and Galinsky (2006) further argue that past successes and the accumulation of rewards can also lead to optimism and risky behavior.

This study combines ideas from Prospect Theory, and the connection between the sense of power and risk that Anderson and Galinsky (2006) observe, to gain insights into decision making under risk. This study expands the scope of the literature on psychological power and decision making under risk by placing the effects of power in the Prospect Theory framework and the role it plays in expected value decisions. More specifically, we tested 60 participants to investigate whether individuals with a heightened sense of power will be less risk averse than individuals without a heightened sense of power. We deviate from Anderson and Galinsky (2006) in that our study provides an economic context with the hypothetical questions as well as novel methodology. Overall, the group that was primed with power chose the prospect with a lower expected value more often than the neutral group.

A. Theoretical Framework

Kahneman and Tversky (1979) formulated a mathematical model of Prospect Theory where the value of a prospect, designated as (V), is conveyed by a decision weight function, $\pi(p)$, with the probability (p), and a value function $v(x)$, with the value of the prospect (x).

$$V(x, p) = \pi(p)v(x) \quad (1)$$

They explain that the decision weight $\pi(p)$ is an individual's assessment of a probability, while the value function (v) is their assessment of the potential outcome of a prospect. Decision makers view risky prospects as a combination of both the weighting function and the value function. They argue that the decision weight will be perceived as smaller than the actual probability of a prospect, $\pi(p) < p$ for small probabilities. Furthermore, decision weights include not only the perceived likelihood of an event, but also the attractiveness of a prospect. After this evaluation phase, individuals choose the prospect with the highest value (Kahneman and Tversky 1979).

II. Materials and Methods

To examine the hypothesized relationship between a heightened sense of power and risk in an economic context, we tested 63 undergraduates and faculty at Eastern Oregon University (28 males and 35 females), most of whom received course credit. The average age was 23 years ($SD=5.73$) and the average years of education was 14.94 years ($SD=1.72$). The first three participants, which included two males and one female were used as a trial run. There were four on campus courses that offered extra credit to students that volunteered to participate, including an introductory chemistry, economics, psychology, as well as an organic chemistry course. The type of courses offering extra credit may have influenced participation. For instance, those in the psychology course may have a

greater interest in participating in a psychological experiment than those in chemistry courses.

Participants were randomly placed in two groups, a high-power (Group A) and a neutral group (Group B). Randomization was accomplished by alternating the group each participant was placed in based on the order they arrived at the study. All sessions were conducted individually by the same author and took approximately 15 minutes. The neutral group (Group B), was asked to read a list of words out loud that may be considered neutral words, including: nonpartisan, table, family, unbiased, tree, uninvolved, computer, food, impartial, neutral, chair, house, noise, toy, and sandwich. Following this, participants were asked to recall a normal day and explain how they felt during that time. Participants then read a short story emphasizing normal daily activity (see Table B.1 in Appendix B). Additionally, there was a small poster hanging on the wall in the room of the experiment was conducted which read, “It Was a Normal Day, Nothing out of the Ordinary Happened that Day,” in large letters.

The high-power group (Group A) was given a list of words to read out loud that may be considered “power” words, including: power, capability, potential, talent, influence, capacity, skill, aptitude, control, leadership, force, prestige, domination, superior, and strength. Participants in Group A were then asked to recall a time they felt powerful, which “was defined... as a situation in which they controlled the ability of another person to get something they wanted, or were in a position to evaluate those individuals” (Anderson and Galinsky 2006, p. 517) and explain how they felt during that time. They also read a short hypothetical story out loud about themselves with a heightened sense of power (see Table B.2 in Appendix B). Asking participants to read a hypothetical story about themselves is a novel approach to priming subjects with power. Additionally, there was a small poster hanging on the wall in the room which read, “The Way to Have Power is to Take it,” in large letters—also a novel method used to improve the odds of priming participants with power using a subliminal

message.

Participants then answered eight hypothetical questions (see Appendix C) that required them to choose between a risky alternative and an alternative with a certain outcome after receiving an initial monetary endowment.

The first three participants were part of a trial run. The confidence in answer (CiA) scale was added after the trial run, which asked each participant to rate on a scale from 1 to 10 how confident he or she was in their choice for each question. At the end of each session, participants were asked to rate their sense of power on a scale from 1 to 10, one being a very low sense of power and ten being very high. To debrief participants, they were offered the opportunity to receive an email explaining the procedure and results. The names as well as any information gathered from participants remain anonymous.

III. Results and Discussion

The results of a chi-square test indicate that there was no statistically significant difference between group A and B for questions 1, 3, 5, 6, and 7, while questions 2, 4 and 8 maintained the hypothesized relationship between power and risk-taking. While questions 2, 4, and 8 were the only questions with a significant difference, they differed from the others in a consistent way (see Table A.1 in Appendix A). These three questions had negative expected values associated with the risk bearing portion of the question,¹ while the risk-bearing portion of the other questions had an expected value of zero. These three questions also had unequal probabilities for gains/losses assigned to their outcomes, while the other questions had a 50 percent chance of a gain and a 50 percent chance of an equal loss. Lastly, the three risk-bearing choices favored by Group A tended to have larger potential gains than the remaining questions.

Why would people in the power-primed group opt for a potential gain with a negative expected value more than people

¹In the case of question 8 we are considering choice “c”.

in the neutral group? One possibility is that powerful people are more confident in their skills/abilities. However, following Kahneman and Tversky (1979) we assume that because probabilities were given, experimental participants adopted these probabilities. We also found no evidence for increased confidence among the power-primed people based on their reported CiA-levels.

Alternatively, Anderson and Berdhal (2002) describe an approach/inhibition theory where powerful people are “more attentive to rewarding aspects of the social environment” (p. 1365). Later, Anderson and Galinsky (2006) found a link between power and risk-taking. They considered three possibilities, powerful people may 1) succumb to the lure of potential rewards without a deliberative calculation of probabilities, 2) optimistically calculate expected values, or 3) be confident in their abilities to capture upside, or deal with negative consequences. The evidence from our experiment is most consistent with the first possibility—the idea that power-primed people focus more on potential rewards. This explains why, when the potential reward was greater than the loss, power-primed people were swayed towards the rewards despite the unfavorable probabilities.

Importantly, the consequence of making choices in this manner was “sub-optimal” in an economic sense. Consequently, we would label it a “bias”.

How might power lead people to make sub-optimal decisions? Kahneman (2013) provides one explanation (see also Thaler and Sunstein 2009). First, power leads a person to focus on a high but unlikely reward. The powerful person works to adjust the value of this high reward given its low probability. As this discounting process takes place, a person is getting closer and closer to the accurate estimation of the prospect’s expected value. However, the “adjustment typically ends prematurely because people stop when they are no longer certain that they should move farther” (Kahneman, 2013, p. 120). Thus, we have a mechanism linking power to suboptimal economic decisions—an “anchoring bias”.

Kahneman and Tversky (1979) acknowledge the potential need for other variables in their decision weighting and value functions. Following the results obtained from Anderson and Galinsky (2006) in addition to our own results, we propose that the value function (v), which is their assessment of the potential outcome of a prospect, is positively related to power (ρ) (see equations 2 and 3) in a way that magnifies the difference between large and small values.

$$V(x, p, \rho) = \pi(p)v(x, \rho) \quad (2)$$

$$\frac{\delta v}{\delta \rho} > 0 \quad (3)$$

IV. Conclusion

The results in this study support Anderson’s and Galinsky’s (2006) finding that a heightened sense of power influences decision making under risk. We put this concept into an economic context and find evidence that power priming can lead to a focus on outsized but low probability rewards. We argue that this outsized reward anchors the power-primed person, who then falls short when adjusting expectations downward based on a low probability. Consequently, we find that there is a tendency for power-primed people to make suboptimal economic choices more often than the neutral group.

The most significant weakness of our experiment is its limited scope. Moreover, the experimental questions may not have been independent of each other—preceding questions may have led people to give a certain answer to the following question. Randomizing the question order may solve this problem. Additionally, we did not measure whether we instilled a “sense of power” from each participant. The only indication of the sense of power they had was from the results of the experiment. Future studies could test the level of psychological power of each participant after they are primed and before they answer questions. Furthermore, the questions did not have anything to do with anyone aside from the person taking the survey, so

it makes it difficult to extend the idea to a situation where someone might be more risk averse because the consequences of a negative outcome could influence other people. Future studies would also need to control for gender differences. We look forward to conducting additional experiments to test the robustness of our findings.

V. Acknowledgements

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References

- [1] Anderson, C., & Berdahl, J.L. (2002) The experience of power: Examining the effects of power on approach and inhibition tendencies. *Journal of Personality and Social Psychology*, 83(6), 1362-1377.
- [2] Anderson, C., & Galinsky, A.D. (2006) Power, optimism, and risk-taking. *European Journal of Social Psychology*, 36(4), 511-536.
- [3] Barberis, N. Huang, M., & Santos, S. (2001) Prospect theory and asset prices. *The Quarterly Journal of Economics*, 116(1), 1-53.
- [4] Bernoulli, D. (1954). Exposition of a new theory on the measurement of risk. *Econometrica*, 22(1), 23–36.
- [5] Carney, D.R., Cuddy, A.J.C., & Yap, A.J. (2010) Power posing: Brief nonverbal displays affect neuroendocrine levels and risk tolerance. *Psychological Science*, 21(10), 1363-1368.

- [6] Cuddy, A.J.C., Wilmuth, C.A., & Carney, D.R. (2012) The benefit of power posing before a high-stakes social evaluation. *Harvard Business School Working Paper*, 13-27.
- [7] Emerson, R. M. (1962). Power-dependence relations. *American Sociological Review*, 27(1), 31-41.
- [8] Galinsky, A. D., Gruenfeld, D. H., & Magee, J. C. (2003). From power to action. *Journal of Personality and Social Psychology*, 85(3) 453-466.
- [9] Gruenfeld, D. H., Inesi, M. E., Magee, J. C., & Galinsky, A. D. (2008). Power and the objectification of social targets. *Journal of Personality and Social Psychology*, 95(1) 111-127.
- [10] Inesi, M. E (2010). Power and loss aversion. *Organizational Behavior and Human Decision Processes*, 112(1), 58-69.
- [11] Kahneman, D. & Tversky, A. (March 1979) Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2), 263-292
- [12] Kahneman, D. (2013) *Thinking fast and slow*. New York, NY: Farrar, Straus and Giroux.
- [13] Keltner, D., Gruenfeld, D. H., & Anderson, C. (2003). Power, approach, and inhibition. *Psychological Review*, 110(2), 265-284.
- [14] Magee, J. C., Galinsky, A. D., & Gruenfeld, D. H. (2007). Power, propensity to negotiate, and moving first in competitive interactions. *Personality and Social Psychology Bulletin*, 33(2) 200-212.
- [15] Rucker, D.D., Galinsky, A., & Dubois, D. (2012) Power and consumer behavior: how power shapes who and what consumers value. *Journal of Consumer Psychology*, 22(3), 352-368.

- [16] Smith, P. K., & Trope, Y. (2006) You focus on the forest when you're in charge of the trees: Power priming and abstract information processing. *Journal of Personality and Social Psychology*, 90(4) 578-596.
- [17] Sturm, R.E., Antonakis, J. (2015) Interpersonal power: A review, critique, and research agenda. *Journal of Management*, 41(1), 136-163.
- [18] Thibaut, J. W., & Kelley, H. H. (1959). *The social psychology of groups*. New York, NY: Wiley.
- [19] Thaler, R. H. & Sustein, C. R. (2009) *Nudge, improving decisions about health, wealth, and happiness*. New York, NY: Penguin Books.
- [20] Thaler, R. H. & Johnson, E. J. (June 1990) Gambling with the house money and trying to break even: The effects of prior outcomes on risky choice, *Management Science*, 36(6), 643-660.

Appendix

Appendix A: Results

Table A.1: Summary Results

Choice after initial endowment		Group A / Group B choosing risky or optimistic option, χ^2
1	Participate or not participate in gamble with 50% chance of winning \$20 and 50% chance of losing \$20.	11/15 $\chi^2_{(1,n=60)} = 1.09,$ $p > .10$
2	Accept \$50 or participate in gamble with 25% chance of winning \$500 and 75% chance of losing \$175.	10/2 $\chi^2_{(1,n=60)} = 6.67,$ $p < .01$
3	After participation in a gamble with 50% chance of winning \$20 and 50% chance of losing \$20, the expected gain is a. \$20, b. \$10, c. \$0, d. \$40. ¹	17/14 $\chi^2_{(1,n=60)} = 0.06,$ $p > .10$
4	Accept \$50 or participate in gamble with 25% chance of winning \$500 and 75% chance of losing \$200.	6/0 $\chi^2_{(1,n=60)} = 6.67,$ $p < .01$
5	After investing you have a 50% chance of winning \$30 and 50% chance of losing \$30, the expected gain is a. \$60, b. \$30, c. \$0, d. \$15. ¹	18/19 $\chi^2_{(1,n=60)} = 0.07,$ $p > .10$
6	Participate or not participate in gamble with 50% chance of winning \$70 and 50% chance of losing \$70.	12/8 $\chi^2_{(1,n=60)} = 1.20,$ $p > .10$
7	Purchase or not purchase 100 shares of a company that has a 50% chance of making \$500 and a 50% chance of losing \$500.	21/16 $\chi^2_{(1,n=60)} = 1.76,$ $p > .10$
8	After a Wall Street deal that earned \$50,000 you can: a. Put the money in the bank and earn \$250 per year, b. Invest in stocks with a 70% chance of a return of \$10,000 per year and a 30% chance you lose \$15,000, or c. Invest the money in stocks which has a 75% chance to give you a return of \$10,000 and a 25% chance you lose \$50,000.	5/1 $\chi^2_{(1,n=60)} = 2.96,$ $p < .10$

¹. Coded as choices a, b and d = a while choice c = b.

Appendix B: Stories for Semantic Priming

Figure B.1: Group B Short Story

I woke up at the usual hour, 8:00am, when I got up, brushed my teeth, and then went to the kitchen to make breakfast. I made the usual breakfast, eggs, ham, and toast with a large glass of orange juice. Following this, I got ready for the day, which to me, seemed like a normal day. Nothing out of the ordinary happened that day.

Figure B.2: Group A Short Story

You are a high-power executive working on Wall Street. You are excellent at your job and have never had any problems getting things done in life. You are successful, and know what you want. Your peers admire you and look to you for answers when they need guidance. You have always been very confident and capable of doing anything you put your mind to. In a way, you could be thought of as a powerful person in general because of your unique qualities that enable you to thrive in everything you do.

Appendix C: Hypothetical Questions on Expected Value

Figure C.1: Question 1

Suppose you gamble and win \$50 right now. You are then given the opportunity to gamble again with a 50% chance to win \$20 and a 50% chance to lose \$20, do you take this bet?

- a. Yes (Expected Group A choice)
- b. No (Expected Group B choice)

Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10

(Mean value of CiA score for Group A is 8.23, while the mean value for Group B is 8).

Expected Value: The expected value of this gamble is $\$(0.5 \times \$20 = 10) + (0.5 \times -\$20 = -10) = \$0$.

Result: The result of the first question suggest that the sense of power did not have a significant influence on risk taking, $\chi^2_{1,n=60} = 1.086, p > .05$.

Figure C.2: Question 2

Suppose you just won \$200 gambling at a casino. You are then offered a chance to make a bet with two options. Option A will give you a sure gain of \$50. Option B has a 25% chance of gaining \$500 and a 75% of losing \$175. What do you do?

- Take the 25% chance of gaining \$500 (Expected Group A choice).
- Take the sure gain of \$50 (Expected Group B choice)

Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10

(Mean value of CiA score for Group A is 7.83, while the mean value for Group B is 8.56).

Expected Value: The expected value of this gamble is $\$0(0.25 \times \$500 = \$125) + (0.75 \times -\$175 = -\$131.25) = -\6.25 .

Result: The result of the second question suggest that the sense of power had a significant influence on risk taking, $\chi^2_{1,n=60} = 6.6667, p < .01$.

Figure C.3: Question 3

Suppose you go into a casino and gamble and win \$50 right now. You then discover another opportunity to gamble with a 50% chance to win \$20 and a 50% chance to lose \$20. What do you expect to gain?

- \$20 (Expected Group A choice)
- \$10 (Expected Group A choice)
- \$0 (Expected Group B choice)
- \$40 (Expected Group A choice)

Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10

(Mean value of CiA score for Group A is 6.93, while the mean value for Group B is 7.06).

Expected Value: $(0.5 \times \$20) + (0.5 \times -\$20) = \$0$.

Result: The result of the third question suggest that the sense of power did not have a significant influence on risk taking, $\chi^2_{1,n=60} = 0.6007, p > .05$.

Figure C.4: Question 4

You are offered an opportunity to gamble with a sure gain of \$50; you also have the option to make bet with a 25% chance of gaining \$500 and a 75% of losing \$200. What do you do? a. Take the 25% chance of gaining \$500 (Expected Group A choice).

b. Take the sure gain of \$50 (Expected Group B choice)

Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10

(Mean value of CiA score for Group A is 8.33, while the mean value for Group B is 8.17).

Expected Value: The expected value of this gamble is $\$0(0.25 \times \$500 = \$125) + (0.75 \times \$ - 200 = -\$150) = -\$25$.

Result: The result of the fourth question suggest that the sense of power had a significant influence on risk taking, $\chi^2_{1,n=60} = 6.6667, p < .01$ Group A should choose B. Take the 25% chance of gaining \$500 (risky choice). Group B should choose A. Take the sure gain of \$50 (risk averse choice).

Figure C.5: Question 5

Suppose you just made \$300 in the stock market. You are then given the opportunity to invest in another stock that has a 50% chance to increase in value, in which case you will gain \$30; it also has a 50% chance to decrease in value, in which case you lose \$30. What do you expect to make from this potential stock purchase? a. \$60 (Expected Group A choice)

b. \$30 (Expected Group A choice)

c. \$0 (Expected Group B choice)

d. \$15 (Expected Group A choice)

Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10

(Mean value of CiA score for Group A is 6.68, while the mean value for Group B is 6.67).

Expected Value: $(0.5 \times \$30 = \$15) + (0.5 \times -\$30 = -\$15) = \$0$.

Result: The result of the fifth question suggest that the sense of power did not have a significant influence on risk taking, $\chi^2_{1,n=60} = 0.0705, p > .05$. Additionally, although question five did not have statistical significant, it should be noted that both Group A and Group B chose the risky choice over the safe choice (Group A chose a risky option 60 percent of the time, while Group B chose a risky option 63.3 percent of the time).

Figure C.6: Question 6

Suppose you are given \$100 right now. You are then given the opportunity to gamble with a 50% chance to win \$70 and a 50% chance to lose \$70. Do you take this bet? a. Yes (Expected Group A choice)
b. No (Expected Group B choice)
Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10
(Mean value of CiA score for Group A is 8.47, while the mean value for Group B is 8.76).
Expected Value: $(0.5 \times \$70 = \$35) + (0.5 \times -\$70 = -\$35) = \$0$.
Result: The result of the sixth question suggest that the sense of power did not have a significant influence on risk taking, $\chi^2_{1,n=60} = 0.273322, p > .05$.

Figure C.7: Question 7

Suppose you purchase 100 shares in Google and your stocks go up 20% making you a profit of \$10,000. Now that you have that additional \$10,000 you have the option to purchase 100 shares of a company that is a 50% chance of increasing in value, in which case you would make \$500 and a 50% chance of decreasing in value, in which case you would lose \$500. Do you purchase these stocks? a. Yes (Expected Group A choice)
b. No (Expected Group B choice)
Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10
(Mean value of CiA score for Group A is 8.03, while the mean value for Group B is 8).
Expected Value: $(0.5 \times \$70 = \$35) + (0.5 \times -\$70 = -\$35) = \$0$.
Result: The result of the seventh question suggest that the sense of power did not have a significant influence on risk taking, $\chi^2_{1,n=60} = 1.7626, p > .05$.

Figure C.8: Question 8

You are a banker on Wall Street and you recently made \$50,000 in a deal in which you did practically nothing. You have three options to invest the money. a. Put the money in the bank which will give you an extra \$250 per year you leave it in there, with 0% chance of losing any of it. b. Invest the money in stocks which has a 70% chance to give you a return of \$10,000 per year with a 30% chance you lose \$15,000. c. Invest the money in stocks which has a 75% chance to give you a return of \$10,000 and a 25% chance you lose all the money you just made in the deal (Expected Group A choice)

Confidence in Answer (CiA): 1 2 3 4 5 6 7 8 9 10

(Mean value of CiA score for Group A is 8.1, while the mean value for Group B is 8.37).

Expected Value: a: $(1 \times \$250) = \250 . b: $(0.6 \times \$10,000 = \$6000) + (0.4 \times -\$15,000 = -\$6,000) = \$0$. c: $(0.75 \times \$10,000 = \$7,500) + (0.25 \times -\$50,000 = -\$12,500) = -\$5,000$.

Result: The result of the eighth question suggest that the sense of power did not have a significant influence on risk taking, $\chi^2_{1,n=60} = 1.9636, p > .05$.

Appendix D: Consent to Participate in Research

Figure D.1: Sample Consent Form

**Eastern Oregon University
Office for Research and Sponsored Programs**

Consent to Participate in Research

Title of Study: Psychology and Economics

Investigator: Johnny Fulfer

Faculty Sponsor: Scott McConnell

Brief Description of Procedure:
Participants will be recruited from Eastern Oregon University and will include both students and faculty. Participants will be asked to follow the procedures of the experiment and answer a series of questions that relate to psychology and economics.

Risks and Benefits:
There are no risks associated with the participation of this study. The benefits include potential extra credit for a course.

Confidentiality:
All information received during the study about participants will remain anonymous. Following the completion of the study, all information gather about participants will be discarded in an effort to keep information anonymous. Any additional requests from participants regarding confidentiality will be addressed in an orderly fashion.

Compensation (may include course credit):
Compensation may include extra credit for courses taken during the spring term upon the approval of their professor.

Right to Refuse or Withdraw (include statement of participant withdrawal procedure):
Participants have the right to withdrawal from the study; however, it is requested that they give the researcher a 24-hour notice. The participants also have the right to stop the experiment at any point during the study and withdrawal from the experiment.

Consent: Your signature below indicates that you have agreed to volunteer as a research subject, that you understand your rights for withdrawal, and that you will notify the investigator in advance if you are unable to participate for any reason.

Date

Date

Participant's Signature

Investigator's Signature

Notes

Notes

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